Phase Calibration With Fast Switching: Implications for Instrumental Phase Stability

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Consider interferometric observations of a target source at frequency f that are rapidly interspersed with observations of a nearby unresolved[1] calibrator at frequency f_c. The raw interferometer phase measurements for each elementary observation are then

$$\begin{aligned}\theta_{\rm target} =& \phi_t + \phi_i(f) + 2\pi f\tau \\ \theta_{\rm cal} =& \phi_i(f_c) + 2\pi f_c\tau'\end{aligned} \tag{1}$$

where ϕ_t is the intrinsic visibility phase of the target source; ϕ_i is the instrumental phase at the given frequency; and τ, τ' are the atmospheric delays during the target and calibrator observations, respectively. Here thermal noise has been neglected, and all of the phase values are assumed constant during each observation. Scaling the second measurement by f/f_c and subtracting allows much of the atmospheric phase effect to be removed:

$$\phi_{\rm corrected} = \phi_t + \phi_i(f) - (f/f_c)\phi_i(f_c) + 2\pi f\,\delta\tau \tag{2}$$

where $\delta\tau = \tau - \tau'$ is the change in atmospheric delay between target and calibrator observations. This result contains the desired target source visibility and the difference in scaled instrumental phase between the two frequencies, along with the residual atmospheric delay fluctuation.

To determine the instrumental phase difference between the two frequencies, we must carry out a separate observation in which a calibrator is observed at both frequencies; we call this the instrumental sequence, and we call the measurements in (1) the target sequence. The instrumental sequence calibrator may or may not be the same as the target sequence calibrator, but again we assume it to be unresolved. Those observations yield

$$\begin{aligned}\theta_{\rm cal1}(f) =& \phi_{i1}(f) + 2\pi f\tau_1 \\ \theta_{\rm cal1}(f_c) =& \phi_{i1}(f_c) + 2\pi f_c\tau_1'\end{aligned} \tag{3}$$

where subscript 1 distinguishes the values obtained in this observation from the similar values obtained during the target sequence. Scaling and subtracting gives

$$\phi_{\rm calibration} = \phi_{i1}(f) - (f/f_c)\phi_{i1}(f_c) + 2\pi f\,\delta\tau_1, \tag{4}$$

and subtracting this from (2) gives

$$\phi_{\rm calibrated} = \phi_t + \Delta\phi_i(f) - (f/f_c)\,\Delta\phi_i(f_c) + 2\pi f\,(\delta\tau - \delta\tau_1)$$

where $\Delta\phi_i = \phi_i - \phi_{i1}$ is the change in instrumental phase at the given frequency between the target sequence and the instrumental sequence. This result contains the desired target visibility phase plus residuals due to instrumental phase drift and uncorrected atmospheric delay fluctuation.

Application to ALMA

The ALMA telescope is designed to allow the target sequence to be carried out with a cycle time as short as about 10 sec, and we expect that the cycle time will typically be less than 15 sec. For example, a cycle might consist of 10 sec on target, 2 sec on calibrator, and two 1.5-sec transistions between the two. On such time scales, it is important that the instrumental phase variation be negligible compared with the atmospheric phase fluctuation. Within each elementary observation, fluctuations in atmospheric delay or instrumental phase will cause loss of coherence, but that is not considered here. For the instrumental sequence, the cycle time might be shorter because very little antenna motion is needed, but the ALMA electronics specification still allows 1.5 sec for the frequency change. We therefore assume that the cycle

[1] The calibrator need not actually be unresolved, but its intrinsic visibility at f_c should be known. We take that phase to be zero for simplicity.

time will be similar. Equations (1) and (3) show only one cycle, but in practice many such cycles will normally be executed and the results averaged. Therefore, the residual atmospheric delays $\delta\tau, \delta\tau_1$ should be treated statictically, with each characterized by its standard deviation. In view of the similar cycle times, we take them to be the same[2] and given by σ_τ. Because the target sequence and instrumental sequence may be substantially separated in time (perhaps many minutes) and also in angular distance on the sky, the two residuals may also be taken to be independent.

Also because of the time between the target sequence and the instrumental sequence, the instrumental phase drifts $\Delta\phi_i(f)$ and $\Delta\phi_i(f_c)$ cannot be neglected. To keep the instrumental error well below the atmospheric error, we find from (4) that we should have

$$\Delta\phi_i(f) - (f/f_c)\,\Delta\phi_i(f_c) << \sqrt{2}\,2\pi f\,\sigma_\tau. \tag{5}$$

A portion of the instrumental phase variation at the two frequencies should be common, so that the subtraction on the LHS of (5) may be helpful, but the components that contribute most to any phase drift are separate at the two frequencies; therefore, for the purpose of making a worst-case estimate, we assume that the two phase drifts are unrelated and can have either sign. The scaling in (5) makes it appear that the instrumental effect is more sensitive to the drift at the lower of the frequencies. However, in practice the phase variation is larger at higher frequencies; if we express the variation in delay units by defining $\Delta\tau_i(f) = 2\pi\Delta\phi_i(f)/f$, (5) simplifies to

$$\Delta\tau_i(f) - \Delta\tau_i(f_c) << \sqrt{2}\,\sigma_\tau. \tag{6}$$

If we now choose to set a specification limiting the standard deviation σ_i of $\Delta\tau_i$ to the same value for all frequencies, we should use

$$\sqrt{2}\,\sigma_i < \sqrt{2}\,\sigma_i(\max) << \sqrt{2}\,\sigma_\tau$$

so

$$\sigma_i(\max) << \sigma_\tau. \tag{7}$$

Note that here σ_i is the standard deviation of the instrumental phase change at a time difference equal to the interval between the target sequence and the instrumental sequence. Again for the purpose of creating a worst-case estimate, let this be 1000 sec.

From [1], we find that a representative point on the 5th percentile curve[3] is at condensed water depth = 0.73 mm, rms delay = 160 fsec. The latter is the uncorrected zenith value at 300m baseline. The residual rms delay σ_τ after correction is then 100 fsec at 45d elevation for 15 sec cycle time with conservative assumptions for wind speed aloft and structure function exponent. If we require that the instrumental error be less than the atmospheric error for the remaining 95% of the time, then we should set $\sigma_i(\max) = 100\,\text{fsec}$. Since most causes of instrumental phase variation are independent between the two antennas of an interferometer, this corresponds to a standard deviation of 70 fsec for each antenna.

This specification means that under 5th percentile conditions and at 45d elevation the total rms error in the calibrated visibility phase will be $2\pi f \cdot 140\,\text{fsec}$. At $f = 875\,\text{GHz}$, this is 0.77 radian. Even if the instrument were perfect, the rms error would be 0.54 radian. Such accuracies are inadequate for imaging, so high frequency imaging work must rely on self-calibration or similar enhancement techniques during post-detection processing.

Under some circumstances, it is possible to execute the target sequence with $f_c = f$, in which case there is no need for the instrumental sequence. Then the atmospheric residual σ_τ enters the error budget only once, and the instrumental error enters only for the one frequency, so the limit on instrumental drift remains the same. But now σ_i must be determined at a time interval equal to the target sequence cycle time, typically 15 sec, rather than the interval between the sequences, typically several minutes. The actual instrumental error is expected to be far smaller for such short intervals. Neglecting instrumental drift entirely, this gives a total rms error of 70 fsec, or 0.39 radian at 875 GHz. (Note that we are still affected by a loss of coherence due to faster phase fluctuations from both the atmosphere and the instrument.)

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[2] This might not be true if the sources are at substantially different elevations.

[3] This is the point that minimizes the sensitivity loss from extinction and coherence when fast switching is used. If other causes of sensitivity loss were considered, or if WVR corrections were used instead, a 5th percentile point with less water vapor and more rms delay fluctuation would maximize the sensitivity. This would lead to less stringent instrumental stability requirements. Thus, this choice is conservative.

Use of Water Vapor Radiometers

If the atmospheric delay can be corrected via water vapor radiometry, then the need for rapidly interspersed calibrator observations might be eliminated, leading to higher observing efficiency (more time on the target). We then have only the elementary observatons of θ_{target} in (1) and $\theta_{\mathrm{cal1}}(f)$ in (3). Subtracting these leaves an atmospheric error term $2\pi f(\tau - \tau_1)$ rather than $2\pi f(\delta\tau - \delta\tau_1)$; that is, we are left with the difference in atmospheric phase between target and calibrator, rather than just the difference in the within-sequence fluctuations. In this scenario, the target and calibrator observations must be separated in time by much more than 15 sec, and they may also be separated by a substantial angle on the sky, including a change in elevation.

Present water vapor radiometer (WVR) technology does not allow the total delays τ, τ_1 to be derived, but only the variation in delay about some unknown mean value. The mean value varies with air mass and drifts with time, so that differential results become inaccurate when elevation changes significantly and for time intervals greater than a few minutes. Furthermore, in this memo τ, τ_1 measure the effect on the interferometric phase, so they are actually the differences in atmospheric delay between the paths to the two antennas of a baseline. With WVRs, this must be estimated by subtracting the results of separate instruments. For these reasons, it is difficult to rely on WVR measurements alone for removal of phase errors due to atmospheric delay. However, if astronomical phase calibration is abandoned entirely, relying only on self-calibration, then WVR measurements may be effective as the sole means of preventing loss of coherence from short-term atmospheric delay fluctuations.

REFERENCE

[1] L. D'Addario and M. Holdaway, "Joint distribution of atmospheric transparency and phase fluctuations at Chatnantor," in preparation, October 2003.