Simulations of the Effects of 1/f Gain Fluctuations on Measuring Linear Polarization with Linear Feeds on ALMA

M.A. Holdaway
NRAO/Tucson

May 10, 2004

Abstract

We make a simulation which is analogous to the most demanding part of the linear polarization with linear feeds imaging problem, and determine that 1/f gain fluctuations of magnitude 5e-4 in 300 s will probably not prevent us from obtaining the fractional polarization specification of 0.001.

1 Simple POL Background (Even if you've never gotten it before)

The polarization background starts with Cotton (MMA Memo 208), and uses the linearized polarization leakage equations just because that is how most people are accustomed to looking at the problem.

We've got the parallel correlations, which are Stokes I modulated by the polarization signal going up and down with the parallactic angle (χ):

$$XX = g_{1x} g_{2x} (I + Q \sin(2\chi) + U \cos(2\chi))$$

$$YY = g_{1y} g_{2y} (I - Q \sin(2\chi) - U \cos(2\chi)),$$

...and we've got the crossed correlations, which are dominated by the polarization signal (if there IS one), plus leakage from Stokes I (ie, the d terms):

$$XY = g_{1x} g_{2y} ((d_{1x} - d_{2y}*) I - Q \sin(2\chi) + U \cos(2\chi))$$

$$YX = g_{1y} g_{2x} ((-d_{1y} + d_{2x}*) I - Q \sin(2\chi) + U \cos(2\chi)).$$

In the above equations, I is actually the Fourier Transform of the I image evaluated at the $1,2$ baseline, and Q and U are the Fourier transforms of those Stokes images.

2 What we ended up doing for simulations

We started this work in AIPS++ because that is where the 1/f noise simulations are done. In the course of this work, we discovered that AIPS++ can't yet do the full polarization simulations we require for ALMA. We can probably do the POL simulations in classic AIPS (but we need to learn how), import the data into AIPS++ as a MeasurementSet, and corrupt the data with the 1/f gains in AIPS++. Instead of following that course, we've made some quick simulations in AIPS++ which are analogous to the most demanding aspects of the observation of linear polarization with linear feeds. We present the justification for this simulation analog here.

The whole purpose to this exercise is to understand the magnitude of the effect of the 1/f gain fluctuations on polarization imaging. The 1/f gain fluctuations will be an issue for XX and YY because the quantity $\Delta g \cdot I$ will be comparable to Q and U; of in terms of the gain fluctuation specifications and the fractional polarization specification, $5e{-}4{\cdot}I$ is not so different from $.001 \cdot I$ (.001 I being the fractional polarization spec). Now, in interferometry, we usually have errors on individual visibilities which are much LARGER than the final image errors, so it is not unreasonable to hope that in THIS case, we could produce an image with less strict errors than we have on the visibilities. (Even systematic errors like 1/f gain fluctuations will have some averaging down.)

In the XY and YX correlations, $\delta g \cdot d \cdot I$ will be about 0.01 times less (because the d terms are lower), so the gain fluctuations are really not a problem for the measurement of XY and YX to sufficient accuracy.

In order to produce Q and U visibilities, we need to use all of the XX, YY, XY, and YX correlations. For example, we define the intermediate quantities A and B (neglecting for the moment gains and d terms, just to see what the data logic is):

$$A = (XX - YY)/2 = Q \sin(2\chi) + U \cos(2\chi)$$

$$B = (XY + YX)/2 = -Q \sin(2\chi) + U \cos(2\chi).$$

Then

$$(A - B)/2 = Q \sin(2\chi)$$

$$(A + B)/2 = U \cos(2\chi).$$

(By the way, we should probably build some logic into the dynamic scheduler that makes sure you don't observe at a single parallactic angle very close to $n45^\circ$ for integral n, as the SNR of the Q or U determination will go to pot.)

The quantity B can be measured with sufficient accuracy, even when we have 1/f gain errors. The quantity A is the problem. We can simulate an exact mathematical analog to $A = (XX - YY)/2$ by switching to circular polarization and imaging Stokes V: $V = (RR - LL)/2$. In these simulations, the easiest thing to do is to simulate a polarization-free case and look to see how large the spurious polarization signal is, and in the case of zero polarization signal, the analog is exact.

We performed a series of simulations with a point source model, 10 s integrations for one hour. Varying levels of thermal noise were added, and 1/f power spectrum gain fluctuations multiplied the noisy visibilities. The R and L (remember, we are doing an analogous simulation) gain fluctuations for a given antenna were completely uncorrelated, while in reality, the X

and Y gain fluctuations on a given ALMA antenna may be partially correlated, as they will experience the same temperature environment. We assumed that every 20 minutes, a high SNR gain solution could be performed, setting the gains at that moment to 1.0000, and then for the next 20 minutes, the gains would migrate away from 1.0000. We then imaged the corrupted visibilities in both Stokes I and V. Indeed, the 1/f gains result in some flux "leaking through" from I into V. We plot up the polarization error as a function of 10 s visibility noise level in Figure 1. We see here that as the noise is decreased, the image plane polarization error decreases as well, until a threshold is reached (due to the 1/f gain fluctuations), below which we cannot go. That error threshold is more than an order of magnitude lower than the 0.001 fractional polarization specification.

Presumably, a point source model should be the easiest thing to image, as all the visibilities are adding up to tell us about that one pixel in the center of the image. A more extended source should result in less averaging of visibilities and larger errors in fractional polarization. To test out that hypothesis, we simulated a Gaussian of 5 synthesized beams FWHM (ie, an area of about 25 beams) in Stokes I. Averaged over the entire model brightness distribution, the error in fractional polarization was 5e-5, and the extreme values of the fractional polarization error over the bright part of the Gaussian were +/- 2e-4, again far below the 0.001 fractional polarization specification.

3 Provisional Conclusions

It seems that the 1/f gain fluctuations at the level of 5e-4 in 300 s will probably not limit the detection of linear polarization at the 0.001 level (in fractional polarization). If there is doubt remaining in the minds of ASAC members, we should either work to modify AIPS++ to permit these simulations, or we should perform the lacking part of these simulations in classic AIPS and import the visibility data into AIPS++ for the application of 1/f gain fluctuations. Of course, doing this level of simulation, we would also want to see the effects of the d terms and the phase errors. As I believe we are on the verge of getting a "polarization czar", perhaps we should tap this new person to perform these rather extensive tests.

What about other errors which might limit the polarization images? Errors in the d terms translate, to first order, into leakage from the total intensity image into the polarization image, approximately as the mean of the combination of residual d terms (whats left of them after calibration): $d_{x,i} + d^*_{y,j}$ (averaged over all baselines). At the VLA, I found the polarization imaging was limited due to fluctuations in the d terms on timescales ranging from minutes to days. At the ATF, which has linear feeds, the experience has been that the d terms are very stable for months at a time (the argument is that it is just geometry, while the VLA's circular feed d terms have standing waves and electronics as well contributing), so we hope that the ALMA will have very stable d terms which we can solve for accurately. If so, then the ALMA polarization will probably be able to go down to 0.001 in fractional polarization or lower.

Errors in the phases of the gains due to the atmosphere could limit the fractional polarization for snapshots of large complicated objects, but that isn't a very realistic case: if you use a snapshot, you probably have a less complicated object, and if you have a complicated object, you'll probably have longer tracks with more averaging. Fast switching will typically have 20 deg phase errors on each visibility, but the fast switching process, performed about once every 30 s, will randomize those errors pretty well, and they will average down assuming

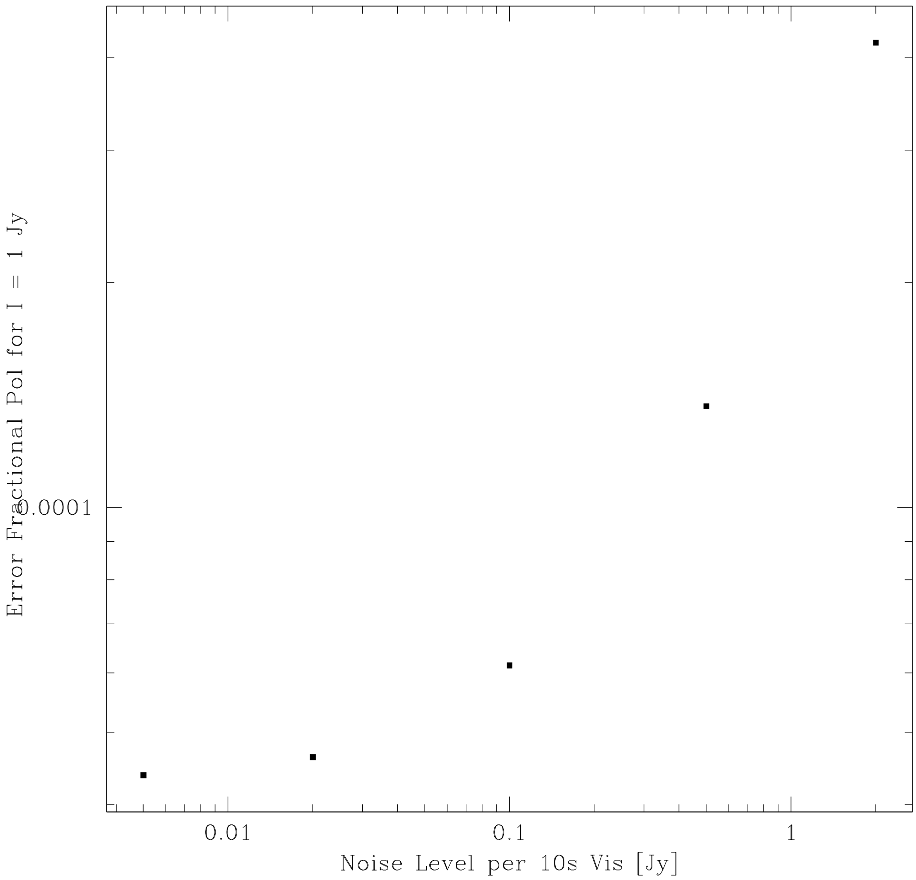

Figure 1: Interaction with thermal noise and 1/f gain errors for a 1 Jy point source: as the thermal noise per 10 s visibility decreases (with the same 1/f gain fluctuations), the image plane polarization error decreases also (ie, follows the noise), but eventually at very low noise, the image plane polarization error is limited by the 1/f gain fluctuations.

there is enough time, or there are enough different baselines contributing to a particular (u,v) cell and the number of image plane pixels we are solving for is not overwhelming.

Cotton, W.D., "Polarization Calibration of the MMA: Circular vs Linear Feeds", MMA Memo 208, 1998.