1. The Crab pulsar has a very steep radio spectral index of approximately $-3$ (i.e. $\nu^{-3}$) over a frequency range from 10 MHz to 10 GHz. If the distance to the Crab nebula is $\sim 2$ kpc, the measured flux density at 400 MHz is 650 mJy, and the spin-down luminosity (i.e. $\dot{E}$) as derived in class is $4 \times 10^{38}$ erg s$^{-1}$, what fraction of $\dot{E}$ does the radio emission account for?

2. Pulsar astronomers parameterize pulsar spin-down in a model-independent way with the relation $\dot{\Omega} \propto \Omega^n$, where $n$ is known as the braking index. Derive a functional form for $n$ in terms of the three observables: $\Omega$, $\dot{\Omega}$, and $\ddot{\Omega}$. What value is $n$ for magnetic dipole radiation?

3. The spin-down relation in terms of period $P$ is $\dot{P} \propto P^{2-n}$.
   (a) Use this relation (assuming an initial spin period $P_0$) to show that the pulsar age $T$ is
   \[
   T = \frac{P}{(n-1)\dot{P}} \left[ 1 - \left( \frac{P_0}{P} \right)^{n-1} \right]
   \]
   without assuming magnetic dipole braking, constant magnetic field strength, or $P_0 \ll P$.
   (b) Show that for the common assumptions of $n = 3$ and $P_0 \ll P$ this reduces to the characteristic age, $\tau_c$, as derived in class.
   (c) If a 100 ms pulsar is found with $\tau_c = 30$ kyr in a supernova remnant where historical or kinematic data suggest a true age of only $\sim 2$ kyr, what does this imply about the pulsar? Can this discovery constrain the braking index? Why or why not?

4. Pulsar dispersion and scattering
   (a) Using the relation that we derived in class for the dispersive delay $t$, derive a simple relation for the total amount of smearing $\Delta t$ in time that will occur over a small frequency bandwidth BW due to uncorrected dispersion. You may assume that the slope of the dispersion curve (i.e. $dt/df$) is constant over the bandwidth in question.
   (b) Estimate the value for the smearing at the central frequency of 1380 MHz for the 3 MHz channels in Figure 6 of the class notes if DM=$100$ pc cm$^{-3}$ and DM=$1000$ pc cm$^{-3}$.
   (c) Above approximately what DM will the above system have too much smearing to detect a 2 ms pulsar?
   (d) Estimate the DM of the pulsar in Figure 6 (which uses the above system) if its spin period is 2 ms, 400 ms, and 8 s.
   (e) Use Figure 9 in the class notes to estimate how pulse scatter-broadening scales as a function of observing frequency $\nu$. Do this using least-squares fitting of measurements from at least 4 or 5 of the profiles. Show a plot of the measurements and the fit and the code that you used to do the fitting.

5. The vast majority of pulsars have been found using Fourier analysis. This problem will make you brush up on your FFT knowledge. For an observation with $N$ samples of duration $dt$ (making a total integration time $T = N \cdot dt$), and a pulsar of spin period $P_{PSR} = 1/f_{PSR}$, where $f_{PSR}$ is the spin frequency:
(a) How many independent Fourier bins will there be in an FFT of the real-valued time series?

(b) What is the Nyquist frequency? What bin number in the FFT does it correspond to?

(c) What is the significance of the zeroth frequency bin?

(d) What is the frequency spacing of the Fourier bins in Hz?

(e) In what Fourier bin will the pulsar’s fundamental (i.e. 1st) harmonic show up? The 2nd harmonic?

(f) Use the various Fourier theorems and relations that you’ve seen in class so far to estimate (do not rigorously derive) how many significant harmonics will be present in the power spectrum if the pulses are roughly Gaussian with fractional width $W/P_{\text{PSR}}$.

6. Pulsar timing

(a) For a 2 ms pulsar where we can measure pulse times-of-arrival (TOAs) to a fractional precision of $10^{-3}$ of the pulsar period, estimate the frequency precision we can achieve over a 10-year span of data using pulsar timing.

(b) Assume we have a binary MSP with TOAs of precision $\sim 1 \mu s$ that is observed (to get 1 TOA) every 15 days. To approximately what precision can we measure the projected semi-major axis of the orbit $x = a \sin i/c$ (usually measured in lt-sec) after 2 years of observations?

(c) Using Kepler’s laws, if the orbital period of the above MSP is 10-days, and assuming that the inclination $i$ is 90°, approximately what is the lowest mass companion star that we can measure using pulsar timing?