1. The Orion Nebula is a nearby $(D \approx 500$ pc) H II region. It is roughly circular in outline, with an angular radius $\theta \approx 10^{-3}$ radians. The associated thermal radio source is called Orion A, and its radio spectrum is shown in Figure 9.1 of the textbook. For low frequencies ($\nu \ll 1$ GHz),

$$\left( \frac{S_\nu}{\text{Jy}} \right) \approx 1000 \left( \frac{\nu}{\text{GHz}} \right)^2.$$ 

The spectrum flattens above $\nu \approx 1$ GHz, and at $\nu = 23$ GHz, $S_\nu \approx 400$ Jy. To simplify your calculations, you may assume that the Orion Nebula (1) is a circular cylinder whose axis lies along the line-of-sight and whose length equals its diameter and (2) has uniform temperature and density throughout.

(a) Estimate the electron temperature of Orion A.

**Solution:** Since the angular radius is $10^{-3}$ radians, the angular diameter, $2\theta = 2 \times 10^{-3}$ radians. Also, $\theta \ll 1$, so $\sin \theta \approx \theta$.

In the optically thick part of the spectrum ($\nu \ll 1$ GHz), $I_\nu = B_\nu(T)$. So we have:

$$\frac{S_\nu}{\pi \theta^2} = \frac{2\nu^2 k T_e}{c^2}$$

$$\frac{1000 \times \nu^2 \times 10^{-18} \times 10^{-26}}{\pi \theta^2} = \frac{2\nu^2 k T_e}{c^2} \quad (\text{all units are SI})$$

$$\Rightarrow T_e = \frac{10^{-41} \times c^2}{2\pi \theta^2 k} = 1.04 \times 10^4 \text{ K}$$

(b) Estimate the emission measure (EM) of Orion A, in units of pc cm$^{-6}$.

**Solution:** At $\nu = 23$ GHz, $S_\nu \approx 400$ Jy.

$$I_\nu = \frac{400 \times 10^{-26}}{\pi \theta^2} = 1.27 \times 10^{-18} \text{ W m}^{-2} \text{ Hz}.$$ 

Since $I_\nu = (1 - e^{-\tau_\nu}) B_\nu$, and $\tau_\nu \ll 1$, we have:

$$I_\nu = \tau_\nu B_\nu$$

$$\Rightarrow \tau_\nu = \frac{I_\nu}{B_\nu} = \frac{I_\nu c^2}{2\nu^2 k T_e} = 7.56 \times 10^{-4}.$$ 

$$\left( \frac{\text{EM}}{\text{pc cm}^{-6}} \right) = \frac{\tau_\nu}{8.235 \times 10^{-2} \left( \frac{T_e}{\text{K}} \right)^{1.35} \left( \frac{\nu}{\text{GHz}} \right)^{2.1}}$$

$$= 1.75 \times 10^6 \text{ pc cm}^{-6}.$$ 

(c) Estimate the electron density in Orion A.

**Solution:** The length of the H II region, $s = 2D\theta = 1$ pc. Since the electron density is uniform in this region,
(d) Estimate the frequency $\nu$ at which the optical depth of Orion A is $\tau = 1$. (The textbook §10.2.1 states that $\tau = 1$ at $\nu = 23.0$ GHz; don’t believe it!)

Solution:

\[
\left( \frac{\nu}{\text{GHz}} \right)^{2.1} (\tau_\nu = 1) = 8.235 \times 10^{-2} \left( \frac{T_e}{\text{K}} \right)^{-1.35} \left( \frac{\text{EM}}{\text{pc cm}^{-6}} \right)
\]

$\Rightarrow \nu = 0.75$ GHz.

(e) Without using the formula for $N_{\text{LyC}}$ in the handout sheet, estimate the ionizing-photon production rate $N_{\text{LyC}}$ needed to keep Orion A ionized.

Solution: The volumetric rate $r_v$ is given by $r_v \approx \alpha_H n_e n_p$. In this case, $n_e = n_p = N_e$.

Therefore, $r_v = 3 \times 10^{-13} \times (1.325 \times 10^4)^2 = 5.26 \times 10^{-07} \text{cm}^{-3} \text{s}^{-1}$. Then,

\[
N_{\text{LyC}} = \pi (D\theta)^2 (2D\theta)r_v = 1.21 \times 10^{49} \text{s}^{-1}.
\]

2. The Earth effectively sits in a low-density H II region made up of the ionized solar wind. The wind has is expanding constantly at $\sim 400 \text{km s}^{-1}$ (i.e. the density decreases as $r^{-2}$) and in the region of the Earth’s orbit, $N_e \sim 10 \text{ cm}^{-3}$. Estimate $\tau$ and $T_b$ at an observing frequency of 100 MHz due to free-free absorption from this wind, at large angles from the Sun.

Solution: Assume that we are observing in the anti-solar direction. Then, the emission measure (EM) is given by

\[
\left( \frac{\text{EM}}{\text{AU cm}^{-6}} \right) = \int_1^\infty \left( \frac{N_e}{\text{cm}^{-3}} \right)^2 \frac{d}{d\left( \frac{s}{\text{AU}} \right)}
\]

where the distances are measured in astronomical units (1 AU = 4.848 $\times 10^{-6}$ pc).

$N_e$ is given by:

\[
\left( \frac{N_e}{\text{cm}^{-3}} \right) = \frac{10}{\left( \frac{s}{\text{AU}} \right)^2}
\]

\[
\left( \frac{\text{EM}}{\text{AU cm}^{-6}} \right) = \int_1^\infty 100 \frac{dx}{x^4} = \frac{100}{3}
\]

$\Rightarrow \text{EM} = 1.61 \times 10^{-4} \text{pc cm}^{-6}$
\[ \tau_{\nu} = 8.235 \times 10^{-2} \left( \frac{T_e}{K} \right)^{-1.35} \left( \frac{\nu}{\text{GHz}} \right)^{-2.1} \left( \frac{\text{EM}}{\text{pc cm}^{-6}} \right) \]

\[ = 8.235 \times 10^{-2} \times (10^5)^{-1.35} \times (0.1)^{-2.1} \times 1.61 \times 10^{-4} \]

\[ \approx 3 \times 10^{-10} \]

To calculate \( T_b \):

\[ T_b = T_e \left( 1 - e^{-\tau} \right) \]

\[ \approx 10^5 \times 3 \times 10^{-10} \]

\[ = 3 \times 10^{-5} \text{ K} \]