Magnetobremsstrahlung

Larmor's formula indicates that accelerating a charged particle produces electromagnetic radiation. In astrophysical situations the strongest accelerations of charged particles are produced by electromagnetic forces. Acceleration by an electric field accounts for free-free radiation. Acceleration by a magnetic field produces ***magnetobremsstrahlung***, the German word for "magnetic braking radiation." Light charged particles (electrons, and positrons if any are present) are more easily accelerated than the more massive protons and heavier ions, so electrons (and possibly positrons) produce virtually all of the radiation observed. The character of this radiation depends on the speed of the electrons, and these somewhat different types of radiation are given specific names. ***Gyro radiation*** is produced by electrons whose velocities are much smaller than the speed of light: $v \ll c$. Mildly relativistic electrons(kinetic energies comparable with rest mass $\times c^2$) emit ***cyclotron radiation***, and ultrarelativistic electrons (kinetic energies $\gg$ rest mass $\times c^2$) produce ***synchrotron radiation***. Synchrotron radiation accounts for most of the radio emission from ***active galactic nuclei*** (AGNs) in galaxies and quasars. It also dominates the low-frequency ($\nu < 30$ GHz) emission from normal star-forming galaxies like our own.

Gyro Radiation

Larmor's equation is nonrelativistic, so we first consider gyro radiation by a particle with $v \ll c$. The ***magnetic force*** $\vec{F}$ on a particle with charge e moving with velocity $v \ll c$ is given by ***Ampere's law***:

$$\boxed{\vec{F} = \frac{e(\vec{v} \times \vec{B})}{c}} \qquad (5A1)$$

The magnetic force is perpendicular to the particle velocity: $\vec{F} \cdot \vec{v} = 0$. No power is transferred to the charged particle and its kinetic energy $mv^2/2$ remains constant. Also, the component of velocity $v_\parallel$ parallel to the magnetic field doesn't change. Since both $|v|$ and $v_\parallel$ are constant, the magnitude of the component of velocity $|v_\perp|$ perpendicular to the magnetic field must also be constant. In a constant magnetic field, the particle moves along the magnetic field line on a uniform helical path with constant linear and angular speeds. In the inertial frame moving with velocity $v_\parallel$, the particle orbits in a circle perpendicular to the magnetic field with the angular velocity ω needed to balance the centripetal and magnetic forces.

$$m|\dot{v}| = m\omega^2 R = |\vec{F}| \ ,$$

where R is the radius of the circle.

$$m\omega^2 R = \frac{e}{c}|\vec{v} \times \vec{B}| = \frac{e}{c}\omega R B$$

$$\omega = \frac{eB}{mc}$$

All nonrelativistic electrons in the same magnetic field orbit at the same frequency and hence emit electromagnetic radiation at the same frequency. We can define the ***gyro frequency*** as

$$\boxed{\omega_{\rm G} \equiv \frac{eB}{mc}} \qquad (5A2)$$

This equation defines $\omega_{\rm G}$ regardless of v/c. The gyro frequency equals the actual orbital frequency only if $v \ll c$. The ***electron gyro frequency*** is

$$\omega_{\rm G} = \frac{eB}{m_e c} = \frac{4.8 \times 10^{-10} \text{ statcoul} \times B}{9.1 \times 10^{-28} \text{ g} \times 3 \times 10^{10} \text{ cm s}^{-1}} \approx 17.6 \times 10^{6} \text{ rad s}^{-1} \times B \text{ (Gauss)}$$

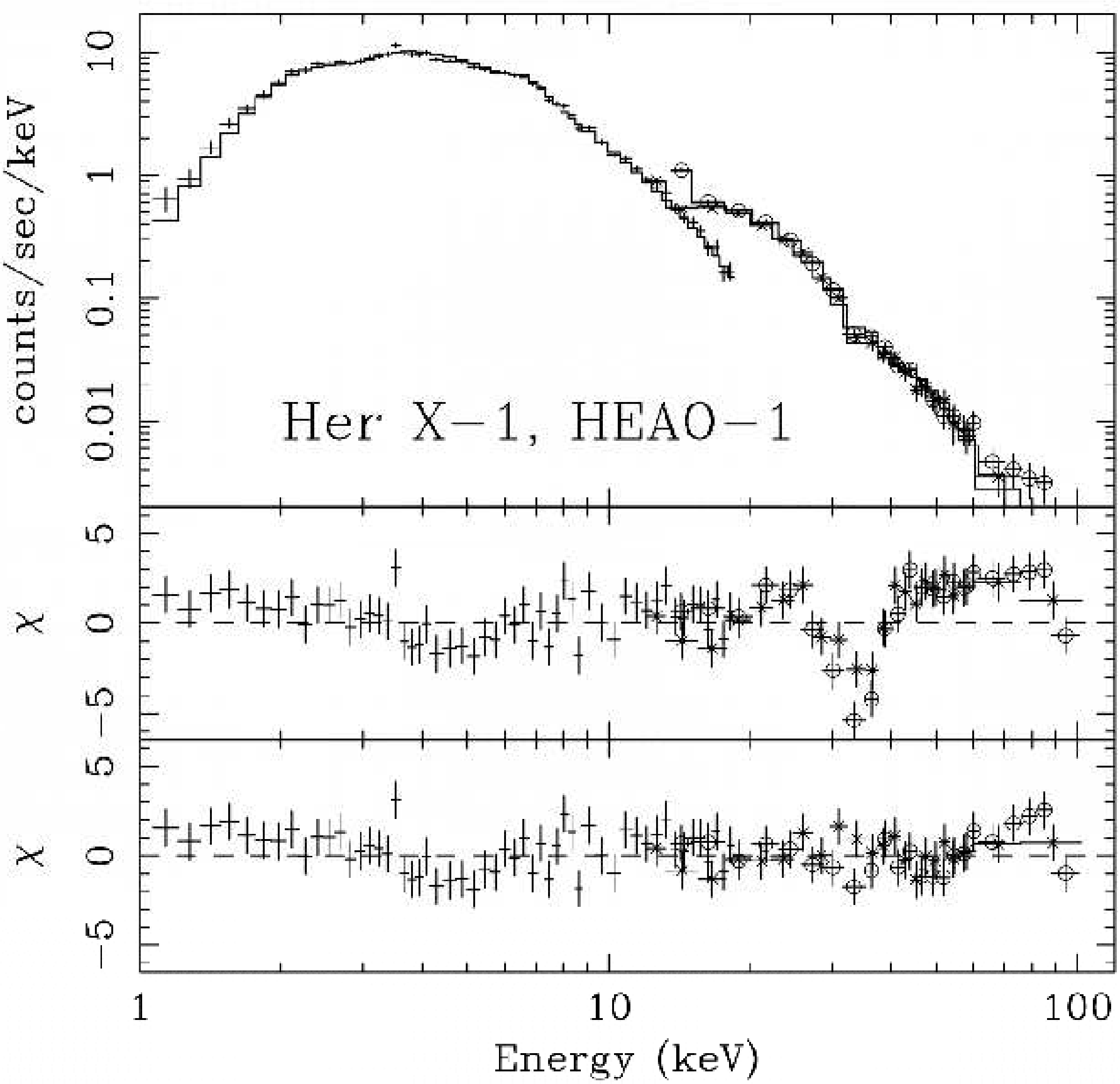
Let $\nu_{\rm G} \equiv \omega_{\rm G}/(2\pi)$ so the electron gyro frequency in MHz is:

$$\boxed{\frac{\nu_{\rm G}}{\rm MHz} = 2.8 \left(\frac{B}{\rm Gauss} \right)} \qquad (5A3)$$

Example: What is the electron gyro frequency in the interstellar magnetic field of our Galaxy, $B \approx 5\,\mu$G?

$$\nu_{\rm G} = 2.8 \text{ MHz} \times 5 \times 10^{-6} \text{ Gauss} = 14 \text{ Hz}$$

Since Larmor's formula states that the radiation field varies with the same frequency as the acceleration, this interstellar gyro frequency is far too low to yield observable radio emission. Gyro radiation is observable only in very strong magnetic fields. An extreme astrophysical example is the magnetic field of a neutron star, $B \sim 10^{12}$ Gauss. For example, the binary X-ray source Hercules X-1 exhibits an X-ray absorption line at photon energy $E \approx 34$ keV.



Gyro-resonance absorption line near 34 keV (Gruber et al. 2001, ApJ, 562, 499).

This spectral feature is thought to be a gyro resonance absorption. Therefore the frequency of this absorption line directly measures the magnetic field strength near Her X-1. The observed photon energy corresponds to the frequency

$$\nu = \frac{E}{h} \approx \frac{34 \times 10^3 \text{ eV} \times 1.60 \times 10^{-12} \text{erg eV}^{-1}}{6.63 \times 10^{-27} \text{erg s}} \approx 8.2 \times 10^{18} \text{ Hz}$$

Equating this frequency to the gyro frequency yields the magnetic field near the neutron star:

$$B = \frac{2\pi \nu_{\rm G} m_{\rm e} c}{e}$$

$$B \approx \frac{2\pi \times 8.2 \times 10^{18}\ \mathrm{Hz} \times 9.1 \times 10^{-28}\ \mathrm{g} \times 3 \times 10^{10}\ \mathrm{cm\ s^{-1}}}{4.8 \times 10^{-10}\ \mathrm{statcoul}} \approx 2.9 \times 10^{12}\ \mathrm{Gauss}$$
