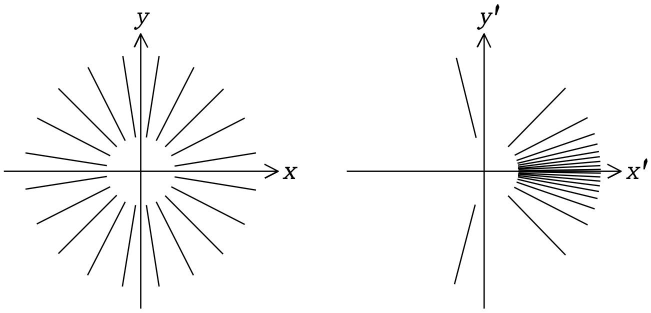
Inverse-Compton Scattering

The total radiation field is normally fairly isotropic in the rest frame of a synchrotron source. However, this radiation field looks extremely anisotropic to the individual ultrarelativistic ($\gamma \gg 1$) electrons producing the synchrotron radiation. Relativistic aberration causes nearly all ambient photons to approach within an angle $\sim \gamma^{-1}$ rad of head-on, as shown in Section 5C. Thomson scattering of this highly anisotropic radiation systematically reduces the electron kinetic energy and converts it into ***inverse-Compton*** (IC) radiation by upscattering radio photons to become optical or X-ray photons. Inverse-Compton "cooling" of the relativistic electrons also limits the maximum rest-frame brightness temperature of an incoherent synchrotron source to $T_{\rm b} \approx 10^{12}$ K.



For a relativistic electron at rest in the "primed" frame moving with velocity v along the x axis, the angle of incidence θ' of incoming photons will be much less than the corresponding angle θ in the rest frame of the observer. This figure shows the aberration of an isotropic radiation field (left) seen in a moving frame with $\gamma = 5$ (right).

IC Power From a Single Electron

To derive the equations describing inverse-Compton scattering, we begin by considering non-relativistic Thomson scattering in the rest frame of an electron. If the Poynting flux (power per unit area) of a plane wave incident on the electron is

$$\vec{S} = \frac{c}{4\pi}\vec{E} \times \vec{H} = \frac{c}{4\pi}|\vec{E}|^2 ,$$

the electric field of the incident radiation will accelerate the electron, and the accelerated electron will in turn emit radiation according to Larmor's equation. The net result is simply to scatter a portion of the incoming radiation with no net transfer of energy between the radiation and the electron. The scattered radiation has power

$$P = |\vec{S}|\sigma_{\rm T} ,$$

where

$$\sigma_{\rm T} \equiv \frac{8\pi}{3}\left(\frac{e^2}{m_{\rm e}c^2}\right)^2 \approx 6.65 \times 10^{-25}\ {\rm cm}^2$$

is called the ***Thomson cross section*** of an electron. In other words, the electron will extract from the incident radiation the amount of power flowing through the area $\sigma_{\rm T}$ and reradiate that power over the doughnut-shaped pattern given by Larmor's equation. The scattered power can be rewritten as

$$\boxed{P = \sigma_{\rm T} c U_{\rm rad}} \qquad (5E1)$$

where $U_{\rm rad} = |\vec{S}|/c$ is the energy density of the incident radiation.

Next consider radiation scattering by an ultrarelativistic electron. The Thomson scattering formula above is valid only in the primed frame instantaneously moving with the electron:

$$P' = \sigma_{\rm T} c U'_{\rm rad}\ .$$

We want to transform this nonrelativistic result to the unprimed rest frame of an observer. We already showed that $P = P'$ so

$$P = \sigma_{\rm T} c U'_{\rm rad}\ .$$

We only need to transform $U'_{\rm rad}$ into $U_{\rm rad}$. Suppose that the electron is moving with speed $v = v_{\rm x}$ in the rest frame of the observer and it is hit successively by two low-energy photons approaching from an angle θ in the observer's frame (θ' in the electron frame) from the x-axis as shown below.

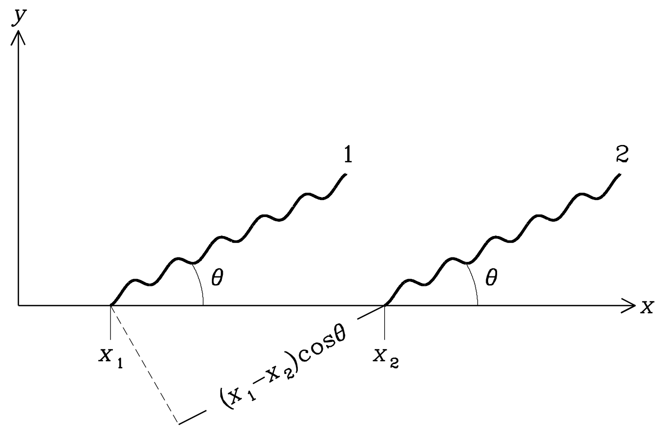
If the coordinates corresponding to the arrival of the first and second photons at the electron (which is always located at $x' = y' = z' = 0$) are

$$(x_1, 0, 0, t_1) \text{ and } (x_2, 0, 0, t_2)$$

in the observer's frame, then the Lorentz transform gives their coordinates as

$$(\gamma v t'_1, 0, 0, \gamma t'_1) \text{ and } (\gamma v t'_2, 0, 0, \gamma t'_2)\ ,$$

also in the observer's frame, as shown below.



Two successive photons striking the moving electron at the angle θ from the x axis, as seen in the observer's frame.

In the observer's frame, the time Δt elapsed between the arrival of these two photons *at the plane normal to the direction of propagation* is

$$\Delta t = t_2 + \frac{(x_2 - x_1)}{c} \cos\theta - t_1$$

$$\Delta t = \gamma t_2' + \frac{(\gamma v t_2' - \gamma v t_1')}{c} \cos\theta - \gamma t_1'$$

$$\Delta t = (t_2' - t_1')[\gamma(1 + \beta \cos\theta)]$$

where $\beta \equiv v/c$.

At this point, we can easily derive the relativistic ***Doppler equation***. Think of the time Δt as the time between the arrivals of two successive cycles of a wave whose frequency is $\nu = (\Delta t)^{-1}$ in the observer's frame and $\nu' = (t_2' - t_1')^{-1}$ in the moving frame. Then

$$\nu^{-1} = (\nu')^{-1}[\gamma(1 + \beta \cos\theta)]$$

or

$$\boxed{\nu' = \nu[\gamma(1 + \beta \cos\theta]} \quad (5E2)$$

In the electron's frame, the frequency ν' (and hence photon energy $E' = h\nu'$) is multiplied by $[\gamma(1 + \beta \cos\theta)]$. Furthermore, the rate at which successive photons arrive is multiplied by the same factor, $[\gamma(1 + \beta \cos\theta)]$. If N is the photon number density in the observer's frame, then $N' = N[\gamma(1 + \beta \cos\theta)]$. In the observer's frame,

$$U_{\rm rad} = N h \nu \ .$$

In the electron's frame

$$U'_{\rm rad} = N'h\nu' = N[\gamma(1+\beta\cos\theta)]h\nu[\gamma(1+\beta\cos\theta)] = U_{\rm rad}[\gamma(1+\beta\cos\theta)]^2 \ .$$

Thus the transformation between $U_{\rm rad}$ and $U'_{\rm rad}$ depends on the angle θ between the direction of the photons and the direction of the electron motion.

The total energy density in the electron frame of a radiation field that is isotropic in the observer's frame is obtained by integrating over all directions:

$$U'_{\rm rad} = \frac{U_{\rm rad}}{4\pi}\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi}[\gamma(1+\beta\cos\theta)]^2\sin\theta d\theta d\phi \ ,$$

where ϕ is the azimuthal angle around the x axis.

$$U'_{\rm rad} = \frac{U_{\rm rad}\gamma^2}{2}\int_{\theta=0}^{\pi}(1+\beta\cos\theta)^2\sin\theta d\theta$$

Evaluating this integral yields

$$U'_{\rm rad} = U_{\rm rad}\left[\frac{4\gamma^2}{3} - \frac{1}{3}\gamma^2(1-\beta^2)\right]$$

Recall that $\gamma^2(1-\beta^2) = 1$ so

$$U'_{\rm rad} = U_{\rm rad}\frac{4(\gamma^2 - 1/4)}{3}$$

Substituting this result for $U'_{\rm rad}$ into

$$P' = P = \sigma_{\rm T} c U'_{\rm rad}$$

yields

$$P = \frac{4}{3}\sigma_{\rm T} c U_{\rm rad}(\gamma^2 - 1/4)$$

This is the total power in the radiation field after inverse-Compton upscattering of low-energy photons. The initial power of these photons was $\sigma_{\rm T} c U_{\rm rad}$, so the *net* power added to the radiation field is

$$P_{\rm IC} = \frac{4}{3}\sigma_{\rm T} c U_{\rm rad}(\gamma^2 - 1/4) - \sigma_{\rm T} c U_{\rm rad}$$

$$P_{\rm IC} = \frac{4}{3}\sigma_{\rm T} c U_{\rm rad}(\gamma^2 - 1)$$

Replacing $(\gamma^2 - 1)$ by $\beta^2\gamma^2$ gives the final result

$$P_{\rm IC} = \frac{4}{3}\sigma_{\rm T} c \beta^2 \gamma^2 U_{\rm rad} \qquad (5E3)$$

for the ***net inverse-Compton power*** gained by the radiation field and lost by the electron. It may be compared with the corresponding synchrotron power

$$P_{\rm syn} = \frac{4}{3}\sigma_{\rm T} c \beta^2 \gamma^2 U_{\rm B}$$

We find the remarkably simple ratio of IC to synchrotron radiation losses:

$$\frac{P_{\rm IC}}{P_{\rm syn}} = \frac{U_{\rm rad}}{U_{\rm B}} \qquad (5E4)$$

Note that synchrotron and inverse-Compton losses both have the same electron-energy dependence ($\propto \gamma^2$), so their effects on radio spectra are indistinguishable.

The IC Spectrum of a Single Electron

What is the spectrum of the inverse-Compton radiation? Suppose the incident radiation field in the observer's frame is isotropic and composed of photons all having the same frequency ν_0, and consider scattering by a single electron moving with ultrarelativistic velocity $+v$ along the x-axis. In the inertial frame moving with the electron, relativistic aberration causes most of the photons to approach nearly head-on. The relativistic Doppler formula gives the frequency ν_0' in the electron frame of a photon approaching near the x axis ($\theta \ll 1$); it is

$$\nu_0' = \nu_0[\gamma(1+\beta\cos\theta)] \approx \nu_0[\gamma(1+\beta)] \ .$$

In the electron frame, Thomson scattering produces radiation with the same frequency as the incident radiation; the scattered photons have $\nu' = \nu_0'$. In the observer's frame, relativistic aberration beams the scattered photons in the direction of the electron's motion. In the observer's frame, the frequency ν of radiation scattered nearly along the $+x$ direction is given by the relativistic Doppler formula:

$$\nu = \nu'[\gamma(1+\beta\cos\theta)] \approx \nu'[\gamma(1+\beta)] \approx \nu_0[\gamma(1+\beta)]^2 \ .$$

In the ultrarelativistic limit $\beta \to 1$,

$$\frac{\nu}{\nu_0} \approx 4\gamma^2 \ .$$

This is the ***maximum frequency*** of the upscattered radiation in the observer's frame.

Oblique collisions ($\theta > 0$) result in lower frequencies ν. For an isotropic radiation field in the observer's frame, the average energy $\langle E \rangle$ of scattered photons is equal to the average power $P_{\rm IC}$ per electron divided by the rate ρ of photon scattering (the number of photons scattered per second by a single electron). This rate is the scattered power divided by the photon energy in the observer's frame:

$$\rho = \frac{\sigma_{\rm T} c U_{\rm rad}}{h\nu_0}$$

Thus

$$\langle E \rangle = h\langle \nu \rangle = \frac{P_{\rm IC}}{\rho} = \frac{4}{3}\sigma_{\rm T} c \beta^2 \gamma^2 U_{\rm rad} \left(\frac{h\nu_0}{\sigma_{\rm T} c U_{\rm rad}} \right)^{-1}$$

The ***average frequency*** $\langle \nu \rangle$ of upscattered photons is

$$\boxed{\frac{\langle \nu \rangle}{\nu_0} = \frac{4}{3}\gamma^2} \qquad (5E5)$$

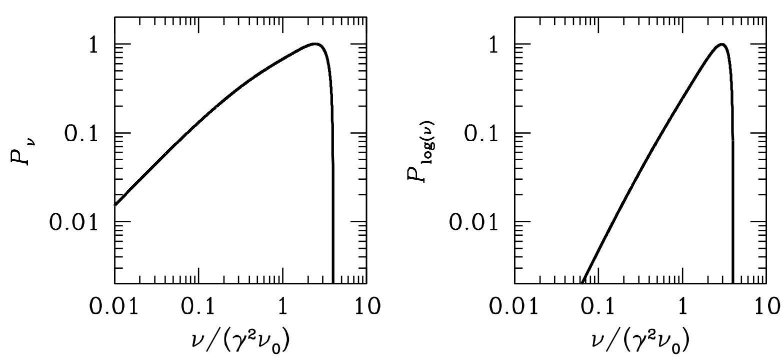
Since the maximum frequency is only three times the average frequency, it is clear that the IC spectrum is sharply peaked near the average frequency.

Example: Radio photons at $\nu_0 = 1$ GHz IC scattered by electrons having $\gamma = 10^3$ will be upscattered to the average frequency

$$\langle \nu \rangle = 10^9 \text{ Hz } \frac{4}{3}(10^3)^2 \approx 1.3 \times 10^{15} \text{ Hz}$$

corresponding to ultraviolet radiation.

The detailed Compton-scattering spectrum resulting from an isotropic and monoenergetic radiation field has been calculated (Blumenthal & Gould 1970, Rev. Mod. Phys., 42, 237; see also Pacholyczyk's *Radio Astrophysics*). It is indeed sharply peaked just below the maximum $\nu/\nu_0 = 4\gamma^2$, as shown in the figure below.



The inverse-Compton spectrum of electrons with energy γ irradiated by photons of frequency ν_0. The log-log plot of power per logarithmic frequency range (right) more accurately shows *how peaked the spectrum is.*

This spectrum is even more peaked than the synchrotron spectrum of monoenergetic electrons. Therefore we don't really need to use the detailed Compton-scattering spectrum of a monoenergetic electrons to calculate the inverse-Compton spectrum of an astrophysical source containing a power-law distribution of electrons. If the electron-energy distribution is $N(E) = KE^{-\delta}$, the scattered spectrum will also be a power law with spectral index $\alpha = (1 - \delta)/2$.

Synchrotron Self-Compton Radiation

Synchrotron self-Compton radiation results from inverse-Compton scattering of synchrotron radiation by the same relativistic electrons responsible that produced the synchrotron radiation. Since

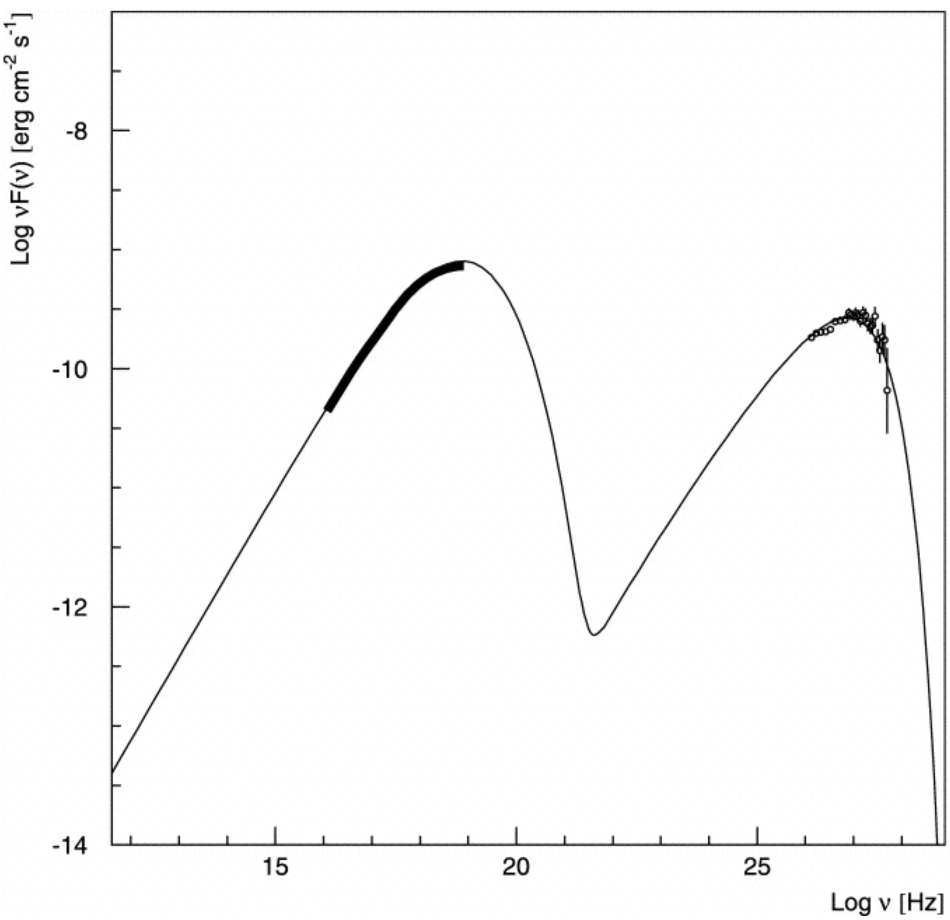
$$\frac{P_{\rm IC}}{P_{\rm syn}} = \frac{U_{\rm rad}}{U_{\rm B}} ,$$

multiplying the density of relativistic electrons by some factor F multiplies the both the synchrotron power and its contribution to $U_{\rm rad}$ by F, so the sychrotron self-Compton power scales as F^2.

The self-Compton radiation also contributes to $U_{\rm rad}$ and leads to significant second-order scattering as the synchrotron self-Compton contribution to $U_{\rm rad}$ approaches the synchrotron contribution. This runaway positive feedback is a very sensitive function of the source brightness temperature, so inverse-Compton losses very strongly cool the relativistic electrons if the source brightness temperature exceeds $T_{\rm b} \sim 10^{12}$ K in the rest frame of the source. Radio sources with brightness temperatures significantly higher than

$$\boxed{T_{\rm max} \sim 10^{12} \text{ K}} \qquad (5E6)$$

in the observer's frame are either Doppler boosted or not incoherent synchrotron sources (e.g., pulsars are coherent radio sources). The active galaxy Markarian 501 emits strong synchrotron self-Compton radiation and the radio emission approaches this ***brightness limit*** for incoherent synchrotron radiation.



The synchrotron (peak near 10^{19} Hz) and synchrotron self-Compton (peak near 10^{27} Hz) spectra of Mkn 501 (Konopelko et al. 2003, ApJ, 597, 851). The ordinate νF_ν on this plot is proportional to flux density per logarithmic frequency range, so the relative heights of the two peaks reflect their relative contributions to $U_{\rm rad}$.
