Self-Consistent Disk Formation, Disk-Envelope Connection, and Global Disk Structure

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Disk Modeling: Various Approaches

- One dimensional, axisymmetric, semi-analytic, $\alpha$-viscosity models: completely ad-hoc treatment of angular momentum, mass transport

- Three dimensional local simulation (periodic shearing box): high resolution possible but still only a local model

- Three dimensional global simulation: limited dynamic range of spatial and temporal scales currently possible

In ALL of these approaches, the disk is usually ISOLATED, and cut off from initial conditions of its parent core and formation process, as well as ongoing interaction with the core envelope!
A New Approach: A Global Cores → Disk Simulation using the Thin-Disk Approximation

Advantages of Thin-Disk Simulation:

- Allows efficient calculation of long-term evolution even with small time stepping, e.g., due to nonuniform mesh. Can study disk accretion for ~ Myr rather than ~$10^3$ yr.
- Can use greater spatial resolution in 2D than in 3D. Study large dynamic range of spatial scales.
- Can run a very large number of simulations – for statistics and parameter study.
A self-consistent model of core collapse leading to protostar and disk formation

Molecular cloud cores are at least mildly nonaxisymmetric.

e.g., Basu & Ciolek (2004) - above

Zoom in to simulate the collapse of a rotating nonaxisymmetric supercritical core

A disk that forms naturally from the collapse of the core. Previous models have usually studied isolated disks.

Our model


Two-dimensional \((r, \phi)\) thin-disk approximation. **Nonaxisymmetric** evolution. Vertical \((z)\) structure is in assumed hydrostatic equilibrium. Sink cell for \(r < 10\) AU after point mass formation.

\[
\frac{\partial \Sigma}{\partial t} = -\nabla_p \cdot (\Sigma \mathbf{v}_p),
\]

\[
\sum \frac{d\mathbf{v}_p}{dt} = -\nabla_p P - \frac{Z}{4\pi} \nabla_r B_z^2 + \frac{B_z B_p}{2\pi} + \sum g_p,
\]

\[
Z = \begin{cases} \frac{c_s^2}{\pi G \Sigma} & \text{if} \quad \Sigma < \Sigma_{cr}, \\ \frac{c_s^2}{\pi G \Sigma_{cr}} & \text{if} \quad \Sigma \geq \Sigma_{cr}, \end{cases}
\]

\[
P = c_s^2 \Sigma + c_s^2 \Sigma_{cr} \left( \frac{\Sigma}{\Sigma_{cr}} \right)^\gamma, \quad \text{barotropic equation}
\]

\[
B_z = \alpha 2 \pi G^{1/2} \Sigma, \quad \text{spatially uniform mass-to-flux ratio; } \mu = \alpha^{-1}
\]

\[
B_p = -\frac{\alpha}{G^{1/2}} g_p,
\]

\[
g_p = -\nabla_p \Phi,
\]

\[
\Phi = -G \int_{r_{\text{out}}}^{r} dr' \int_{0}^{2\pi} \frac{\Sigma(r', \phi') d\phi'}{\sqrt{r'^2 + r^2 - 2r'r \cos(\phi' - \phi)}}, \quad \text{Fast potential solver exists for 2D } (r, \phi) \text{ employing a FFT.}
\]
What’s not included in this model (as of now!)

- Ambipolar diffusion or other non-ideal MHD effects
- Magnetic braking
- Physics of inner disk (~ 10 AU) inside central sink cell
- Magnetorotational instability (can’t occur in thin-disk model)
Core initial conditions

\[
\sum = \frac{r_0 \sum_0}{\sqrt{r^2 + r_0^2}},
\]

\[
\Omega = 2\Omega_0 \left( \frac{r_0}{r} \right)^2 \left[ \sqrt{1 + \left( \frac{r}{r_0} \right)^2} - 1 \right],
\]

\[
B_z = \alpha 2\pi G^{1/2} \sum.
\]

Pick \( r_0, \Omega_0, \alpha \), so that core is mildly gravitationally unstable initially.

Add mild initial nonaxisymmetric \((m=2)\) perturbation.

\[
r_0 = 6.4 \times 10^{-3} \text{ pc} = 1.3 \times 10^3 \text{ AU}
\]

\[
r_{out} = 0.05 \text{ pc} = 10^4 \text{ AU}
\]

\[
\Omega_0 = 1.5 \text{ km s}^{-1} \text{pc}^{-1} = 4.9 \times 10^{-14} \text{ rad s}^{-1}
\]

\[
\alpha = 0 \text{ or } 0.3
\]

These profiles represent best analytic fits to axisymmetric models of magnetically supercritical core collapse (Basu 1997).

All scale as \( r^{-1} \) at large radii.

Basic qualitative results are independent of details of initial profiles.
Self-consistent formation of the protostellar disk and envelope-induced evolution

Evolution of the protostellar disk
Black space is not empty!

Mass infall rate onto the protostar
Spiral structure and protoplanetary embryo formation
Gravitational Torques on Protoplanetary Clumps

Unshaded regions have positive gravitational torque; gray to red regions have negative torque.
Mass accretion bursts and the $Q$-parameter

Black line - mass accretion rate onto the protostar; Red line – the $Q$-parameter

The disk is strongly gravitationally unstable when the bursts occur

\[ Q = \frac{c_s \kappa}{\pi G \sum} \]
Accretion history of young protostars


A Closer Look at Spatial Profiles – Azimuthally Averaged

\[ Q = \frac{c_s K}{\pi G \Sigma} \]

Accretion and instability help to self-regulate disks to a near-uniform \( Q \) distribution

\[ \Sigma \propto r^{-3/2} \]

Vorobyov & Basu (2007)
Conclusions

• Thin-disk simulations can cover large dynamic range of spatial and temporal scales and thereby reveal new physics that is currently inaccessible in 3D.

• Early disk accretion: Infall of envelope material leads to nonlinear gravitational instability in disk $\rightarrow$ formation of dense protoplanetary clumps $\rightarrow$ driven into central protostar due to gravitational torques. This process repeats as long as sufficient envelope material accretes onto disk. A basis for understanding the FU Ori phenomenon.

• Late time accretion: Disk maintains a sharp edge, and evolves due to gravitational torques. Magnitude of accretion rate agrees with observed values. A self-regulated state is set up in which the Toomre parameter remains spatially uniform $\Rightarrow$ surface density scales as $r^{-3/2}$.

• Spatial structure of disk can likely be distinguished from predictions of other physical models – a key task for mm/submm observations.