Very Long Baseline Interferometry and the VLBA
ASP Conference Series, Vol. 82, 1995
J. A. Zensus, P. J. Diamond, and P. J. Napier (eds.)

Printed June 13, 1995

CHAPTER 15

Polarimetry

W. D. COTTON

NATIONAL RADIO ASTRONOMY OBSERVATORY

Abstract Theoretical and practical aspects of imaging VLBI polarization data are discussed, with emphasis on linear polarization measurements.

Contents

15.1 Introduction

The linearly polarized emission from nonthermal radio sources can reveal a great deal about the ordering and orientation of magnetic fields and plasma in and around the source. Interferometric measurements have been made for a number of years using both connected element (Conway and Kronberg 1969; Fomalont and Wright 1974; Noordam 1991; Lüdke 1993) and VLBI techniques (Cotton et al. 1984; Roberts, Brown and Wardle 1991; Cotton 1993; Leppänen, Zensus and Diamond 1995). This lecture discusses the theoretical and practical aspects of calibrating and imaging VLBI polarization data, based on the development in Cotton (1993), and focused on the measurement of linear polarization.

15.2 Instrumental Response

The polarization of a signal is usually determined by measuring the cross correlation of the signals received by a pair of detectors sensitive to orthogonal polarizations. These detectors are called *feeds* for historical reasons. For reasons to be described later, this discussion will focus on measurements made with feeds sensitive to circular polarization. The Stokes parameters, i, q, u and v, of the signal detected are related to the measured cross correlations among circularly polarized feeds by the following relations:

$$i = 0.5 \times (RR + LL) \tag{15.1}$$
$$q = 0.5 \times (RL + LR) \tag{15.2}$$
$$u = 0.5 \times j(LR - RL) \tag{15.3}$$
$$v = 0.5 \times (RR - LL)\,. \tag{15.4}$$

RR, LL, RL, and LR represent the correlations derived from the various combinations of right (R) and left (L) circular feeds and $j = \sqrt{-1}$. RR and LL will be referred to as parallel polarized correlations and RL and LR as cross polarized. In this lecture I, Q, U and V will represent the Stokes' parameters of the derived images and are related to the measured i, q, u, and v values by a Fourier transform relation (Clark 1989). The observed correlations are related to the Stokes' parameters by:

$$RR = i + v \tag{15.5}$$
$$LL = i - v \tag{15.6}$$
$$RL = q + ju \tag{15.7}$$
$$LR = q - ju\,. \tag{15.8}$$

In practice, a variety of instrumental and atmospheric effects corrupt the measured cross correlations. These effects must be determined and removed for the relationships above to hold.

It is beyond the scope of this lecture to review the techniques for the calibration of total intensity observations; the reader is referred the lecture by Moran and Dhawan. The calibration of polarization data does put serious constraints on the total intensity calibration as will be discussed later. In addition, the response of the interferometer to a polarized signal must be measured and the data corrected to remove the imperfections in the instrumental response.

Polarization calibration consists of two distinct steps: 1) determination of the true instrumental feed response and correction of the data, and 2) correction of the apparent polarization angle (the angle of the apparent E-vectors of the polarized radiation on the sky) to the top of the atmosphere (actually the ionosphere).

Calibration of the feed response is necessary because any feed will respond to signals with polarizations other than the nominal one. The response can be modeled in a number of ways, two are presented here: 1) model the feed as sensitive to elliptically polarized radiation, or 2) model the feed as sensitive to the desired polarization plus a complex factor times the orthogonal polarization. These models are equivalent in their full form but the second can be linearized and, when applicable, speeds the computations.

The apparent polarization angle gets corrupted by a number of effects. Phase calibration usually consists of independent calibrations of the parallel system of correlations. These calibrations will be internally consistent but still allow *right-left* phase and delay differences between the two systems. A phase difference between the systems that is constant in time and position results in a constant rotation of the apparent orientation of the derived *E-vectors* on the plane of the sky. Another problem is that the apparent orientation of the E-vectors can be rotated by passage through the magnetized plasma of the ionosphere, owing to Faraday rotation (Pacholczyk 1970). This effect can be quite variable with time and observing geometry and can be expressed as a time variable right-left phase difference.

15.2.1 Interaction with Total Intensity Calibration

Polarization calibration is affected by and places constraints on prior total intensity calibration. Total intensity calibration must be constrained to maintain the inherent phase relationships in the data. Calibration values determined from RR and LL data systems can be corrected for the differences in the two systems and applied to the RL and LR data. For wideband recording systems there is usually the complication that the observed bandpass is broken up into a number of sections with partially independent electronics and cabling.

15.2.1.1 Phase

Phase calibration of instrumental and atmospheric effects can use either observations of the target source (self-calibration) or of a nearby calibrator source of known structure (phase-referencing). The requirements for polarization calibration are similar for these two cases.

Independent calibration of the right and left handed systems allows an arbitrary offset between the two systems. Interferometers only measure phase differences so the phases determined in any given solution are relative to a *reference* antenna. If the phases are referred to the same reference antenna at all times in both systems and the phase difference between the systems is constant at the reference antenna then the phase difference between the parallel systems will have a constant value. A properly calibrated antenna can maintain a constant instrumental phase difference but ionospheric Faraday rotation (discussed in more detail later) can cause significant problems.

The interaction between polarization and phase calibration is illustrated by the effect of parallactic angle. As viewed from the source, a fixed feed on an antenna with an altitude-azimuth (alt-az) mount will appear to rotate in time as the earth rotates. The orientation of the feed is called the parallactic angle. The effect of parallactic angle on feeds sensitive to circular polarization is to rotate the phase of the signal received. For parallel-hand correlations the phase is rotated by the difference in the parallactic angles but for cross polarized correlations the observed phases are rotated by the sum of the parallactic angles.

The parallactic angle is a function of only the observing geometry and can be easily computed. The calibration process is greatly simplified if the parallactic angle effects are removed from the data before any phase or polarization calibration and the following discussion will assume that this has been done. The removal of the parallactic angles must be accounted for in the polarization calibration.

In a similar fashion, modeling the feeds as sensitive to elliptically polarized radiation introduces a phase rotation due to the orientation of the reference antenna. This effect is similar to that for parallactic angle except that it is constant in time (hopefully) but is unknown prior to calibration. If all phase solutions are referred to the same reference antenna then this effect is constant and its removal is absorbed into the right-left phase calibration.

15.2.1.2 Delays and Rates

Delay and rate residuals are the frequency and time derivatives of the phase residual and their determination in a *global fringe fit* is a simple extension of the determination of the phase residual. Delay and rate residuals, like phase residuals, are with respect to a given reference antenna and are usually determined separately for each of the parallel-handed systems. This procedure allows arbitrary offsets in the delay residuals in each of these systems. If each of the antennas involved has a constant (in time) phase relationship between the right and left systems then the rate residuals should be identical. If the phase relationship between the parallel systems is not constant in time at each antenna then the relationship must be known and corrected for the data to be imaged. If the delay residuals are all referred to a common reference antenna there should be a constant offset between the delays in the parallel hand systems.

15.2.2 Ellipticity-Orientation Model

One model is that the feed responds to elliptically polarized signals. Following Fomalont and Wright (1974) the response of a feed to the electric field can be described as:

$$\begin{aligned}\mathbf{G} = \; & \mathbf{e}_x[\cos(\theta)\cos(\phi+\chi) - j\sin(\theta)\sin(\phi+\chi)] \\ & + \mathbf{e}_y[\cos(\theta)\sin(\phi+\chi) + j\sin(\theta)\cos(\phi+\chi)]\,,\end{aligned} \tag{15.9}$$

where $\mathbf{e}_x$ and $\mathbf{e}_y$ are orthogonal unit vectors, θ is the feed ellipticity, ϕ is the orientation of the ellipse, $j = \sqrt{-1}$ and χ is the parallactic angle given for an alt-az mounted antenna: [1]

$$\chi = \tan^{-1}\left(\frac{\cos(lat)\sin(ha)}{\sin(lat)\cos(dec) - \cos(lat)\sin(dec)\cos(ha)}\right)\,, \tag{15.10}$$

where lat is the antenna latitude, dec is the declination of the source and ha is the hour angle of the source.

The response of an amplitude and phase calibrated interferometer, kl, can be written as follows:

$$\begin{aligned}V_{kl}^{obs} = & g_k g_l^* \{RR_{kl}[(\cos\theta_k + \sin\theta_k)e^{-j(\phi_k+\chi_k)}] \times [(\cos\theta_l + \sin\theta_l)e^{j(\phi_l+\chi_l)}] \\ & + RL_{kl}[(\cos\theta_k + \sin\theta_k)e^{-j(\phi_k+\chi_k)}] \times [(\cos\theta_l - \sin\theta_l)e^{-j(\phi_l+\chi_l)}] \\ & + LR_{kl}[(\cos\theta_k - \sin\theta_k)e^{j(\phi_k+\chi_k)}] \times [(\cos\theta_l + \sin\theta_l)e^{j(\phi_l+\chi_l)}] \\ & + LL_{kl}[(\cos\theta_k - \sin\theta_k)e^{j(\phi_k+\chi_k)}] \times [(\cos\theta_l - \sin\theta_l)e^{-j(\phi_l+\chi_2)}]\}\,,\end{aligned} \tag{15.11}$$

where the effects of phase calibration are given by

$$g_R = e^{-j(-\chi-\phi_R+\phi_{Rref})}\,, \tag{15.12}$$

and

$$g_L = e^{j(-\chi-\phi_L+\phi_{Lref}+\phi_{R-L})}, \tag{15.13}$$

$$\phi_{R-L} = \text{right-left phasedifference}\,. \tag{15.14}$$

[1] Actual computation of the parallactic angle should involve a two argument arctangent function to resolve the quadrant ambiguities.

ϕ_{Rref} and ϕ_{Lref} are the orientations of the right and left circular feeds of the reference antenna. RR_{kl}, LL_{kl}, RL_{kl} and LR_{kl} are the amplitude and phase calibrated responses of interferometers with perfect right and left circular feeds to the source polarization. To approximate the effects of source resolution, the values of RR_{kl}, LL_{kl}, RL_{kl} and LR_{kl} can be estimated from the observed total intensity, $0.5(RR_{kl} + LL_{kl})$, and fractional Q, U and V values. This *similarity* approximation is discussed in more detail in a later section.

The relationship above can be expressed in matrix notation as:

$$F_{kl}^{obs} = M_{kl} F_{kl}^{true} , \tag{15.15}$$

where the F_{kl}^{obs} are the observed correlations and the F_{kl}^{true} are the true values (RR_{kl} etc.) and M_{kl} is a matrix whose elements are given above. This relationship can be inverted to determine the corrected Stokes' correlation vector.

15.2.3 Leakage-Term Model

The *leakage* term (Conway and Kronberg 1969) model can be used to parameterize the feed response after total intensity calibration:

$$R_k = G_{kR} e^{j(\chi_k)} (E_R e^{-j\chi_k} + D_{kR} E_L e^{j\chi_k}) \tag{15.16}$$

$$L_k = G_{kL} e^{-j(\chi_k)} (E_L e^{j\chi_k} + D_{kL} E_R e^{-j\chi_k}) , \tag{15.17}$$

where E_R and E_L are the electric field strengths of the right and left hand polarizations, G_{lR} and G_{lL} are the complex amplitude and phase calibration gains of the right and left circularly polarized feeds. The complex D terms represent the *leakage* of the orthogonal polarization into each feed.

In the limit of weakly polarized sources and nearly perfect feeds the second order terms in D and source polarization are small and can be ignored. This leaves a linear approximation for the feed response after application of amplitude and phase calibration:

$$\frac{RL_{kl}^{obs}}{ii} = \frac{(q + ju)}{ii} + D_{kR}\, e^{-2j\chi_k} + D_{lL}^{*}\, e^{-2j\chi_l} \tag{15.18}$$

$$\left(\frac{LR_{kl}^{obs}}{ii}\right)^{*} = \frac{(q + ju)}{ii^{*}} + D_{lR}\, e^{-2j\chi_l} + D_{kL}^{*}\, e^{-2\chi_k} , \tag{15.19}$$

where $ii = 0.5(RR_{kl}^{obs} + LL_{kl}^{obs})$.

This model can be fitted to amplitude and phase calibrated data to determine the two complex parameters D_R and D_L needed to describe each feed pair. A first order correction is then:

$$RL_{kl}^{corr} = RL_{kl}^{obs} - RR_{kl}^{obs} D_{lL}^{*} e^{-2j\chi_l} - LL_{kl}^{obs} D_{kR} e^{-2j\chi_k} \tag{15.20}$$

$$LR_{kl}^{corr} = LR_{kl}^{obs} - RR_{kl}^{obs} D_{kL} e^{2j\chi_k} - LL_{kl}^{obs} D_{lR}^{*} e^{2j\chi_l} . \tag{15.21}$$

This approximation cannot be used to correct the RR and LL data as these corrections depend on higher order terms ignored in the linearized model. If corrections to RR and LL data are needed or the approximation of nearly perfect feeds is inadequate then the linear approximation cannot be used. A fully nonlinear leakage-term model or the ellipticity-orientation model are required in these cases. Unfortunately, the solutions for both of these models is nonlinear.

15.2.4 Polarization Angle

The orientation of the linearly polarized component can be described as the position angle of the apparent *E-vectors* of the radiation. The apparent polarization angle can be modified by Faraday rotation in a magnetized plasma between us and the source and by various instrumental effects. Corrections for Faraday rotation and instrumental effects must be determined from observations of a calibrator of known polarization angle.

15.3 Phase Calibration

This section discusses some of the practical aspects of the phase calibration of polarization data. For observations made with circularly polarized feeds the two parallel-hand systems can be calibrated independently. This phase calibration must be constrained to maintain the relationship of the phase and its derivatives between the two parallel-handed systems, and to reduce this relationship to the differences at a single antenna. Linear and circularly polarized feeds require different treatment.

15.3.1 Linearly Polarized Feeds

Many sources and calibrators have significant linear polarization from a few percent to tens of percent; this is the usual interest in making polarization images. Orthogonal pairs of linear feeds will measure different aspects of the source and will be affected by both the source polarization and feed imperfections. Thus, for observations with linearly polarized feeds, the calibration of the two orthogonal polarizations must be coupled; more critically, the amplitude and phase calibration must be done simultaneously with the determination of the feed polarization parameters. It is beyond the capability of available computing systems to do joint fringe fitting, amplitude calibration, polarization calibration, and source modeling. Use of linear feeds will not be considered further. Mixed linear and circular feeds have similar problems.

15.3.2 Circularly Polarized Feeds

The circular polarization of synchrotron sources is usually very small, a few tenths of a percent or less. In this case, both of the parallel-hand systems of data measure effectively the total intensity and can be calibrated independently.

In addition, to first order, the feed imperfections will not degrade the parallel-hand data and the amplitude and phase calibration can be done independently of the polarization calibration.

It is possible to determine an amplitude and phase (and its time and frequency derivatives) calibration independently for each of the parallel-hand systems before determining the instrumental polarization. The signals at each antenna should have an constant intrinsic phase relationship between the right and left handed systems. This intrinsic relationship must be maintained.

15.3.3 Phase Rereferencing

Interferometers measure differences in phase, delay and rate and the calibration process determines these values with respect to a reference antenna. The requirement of a constant right-left phase (and delay and rate) difference means that all phase-like calibrations should be referred to the same reference antenna. This may require referencing phases to an antenna at times during which it was not observing the source. The phase differences between the primary and any secondary reference antennas can be determined from times at which both antennas were observing the same source and then interpolated/extrapolated to times when only the secondary reference antenna was used. This interpolation/extrapolation should be done in a fashion that maintains R-L differences; i.e., both polarization must have valid solutions to be used to estimate differences between primary and secondary reference antennas. The difference between primary and secondary reference antennas can then be used to correct the calibration to what would have been used for the primary antenna.

15.3.4 Phase-Cals

Multiband VLBI record systems usually incorporate an instrumental phase calibration system, called *phase cals* or *pulse cals* to align the relative phases of the different frequency bands. These calibration systems are also useful in insuring a constant right-left phase relationship. The phase cals consist of one or more tones in each recorded band. The phase of these tones is measured in the recorded signal and used to remove the instrumental phase. Ideally, these tones should be injected into the feeds and thus measure the effects of the entire signal processing path. For phased arrays this is not practical and for these and other antennas the phase cal tones may be injected later into the signal processing path.

If possible, the reference antenna should have phase cal tones injected at or near the feed to remove most or all of the possibly time variable instrumental right-left phase difference. The parallel hand calibration will reference all the other antenna phases to the reference antenna ensuring a constant instrumental right-left phase difference.

15.3.5 Fringe Fitting

Fringe fitting of the parallel-hand systems using circular feeds can be done independently. However, this will leave a phase and delay offset between the two systems. All phases and derivatives of phase should be referred to the same reference antenna to insure that the phase, delay and rate offsets will be constant in time. These offsets between the two systems must be determined and the parallel-hand calibrations adjusted so that they can be applied to the cross hand data. An adaptation of baseline based fringe fitting to polarization data is discussed in Brown, Roberts and Wardle (1989).

15.3.5.1 Parallel-Hand Fringe Fits

The parallel-hand solutions should be fringe fitted in the usual fashion (see lecture on fringe fitting). For multi-band data it is desirable to use the phase cals and align the multi- and single-band delays using a single segment of calibrator data prior to fringe fitting. This will change the right-left phase differences of all antennas to that of the reference antenna used for this calibration reducing the likelihood of problems later. The resulting solutions should be processed in a way to enforce right-left phase stability. This includes the following: the rate residuals of the two parallel systems should be averaged or otherwise forced to be the same. Noise and differences in time sampling can cause differences in the estimated rates. A false difference in the rate corrections will cause a time variable right-left phase difference.

All phase-like solutions should be referred to the same reference antenna. This may require interpolation/extrapolation of the differences between primary and secondary reference antennas. These differences should only be used from times when both of the parallel system solutions are available.

Filtering of solutions, e.g., smoothing in time, should be done jointly for phase and rate as one is the time derivative of the other. Smoothing of the phase and multi-band delays should similarly be coupled. In addition, this filtering must be constrained to retain the phase relationship between the two parallel systems, e.g., use the same rate residual at the same time.

15.3.5.2 Right-left Delay Calibration

Since the electronics and cabling for the signals in the parallel systems may be different, the delay residuals may be different for these two systems. (Also see the lecture on fringe fitting for a discussion on multi- and single-band delays). In general, there will be a difference in the delay residual between the two systems. This right-left delay difference must be determined by examining the cross polarized correlations. Under the conditions of right-left phase coherence at each antenna and all phases, delays and rates referred to the same antenna then the right-left delay differences should be constant. This right-left delay difference can then be determined from any baseline using a short time segment

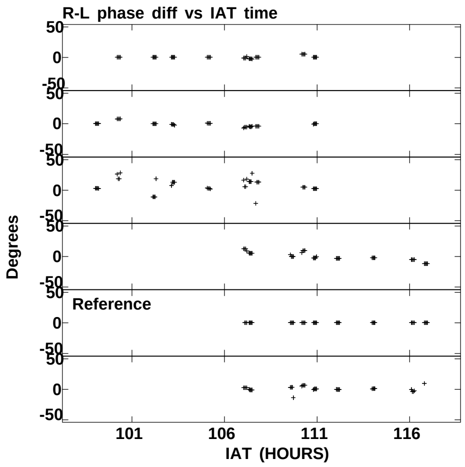

Figure 15.1: This figure shows the difference in the right and left hand phase solutions as a function of time for a number of antennas. Solutions for three different calibrators are shown. The plot labeled "Reference" shows the values for the reference antenna. The solutions have all been referred to this reference antenna.

of calibrator observations with sufficient signal-to-noise ratio after application of the parallel-hand delay, rate and phase calibration. The right-left delay difference can be applied by adding (or subtracting as appropriate) the difference to (from) the solutions for one of the parallel systems and then using the corrected parallel-hand corrections for both the parallel and cross handed data.

15.3.6 Right-Left Coherence

In the absence of significant, variable Faraday rotation and the presence of a properly functioning phase cal system, the phase of the two parallel-hand systems should have a constant offset at each antenna. The phase calibration scheme outlined above should reduce the delay, rate and phase differences between the two parallel-hand systems to the values for the primary reference antenna. Since the phase difference is expected to be constant in time then the right-left fringe rate residual should be zero.

This constant right-left phase difference should be apparent in the adjusted phases in the parallel-hand fringe fit solutions. Figure 15.1 shows the difference in the right and left hand phase corrections for a set of fringe fitting results after they all have been referred to the same reference antenna. The relative consistency indicate that the phases have been properly referenced to the common reference antenna and ionospheric Faraday rotation is not a problem.

Plots like those in figure 15.1 are very useful diagnostics of the success of the phase calibration. In addition, these plots will easily reveal any problems due to variable ionospheric Faraday rotation.

15.3.7 Ionospheric Faraday Rotation

The magnetized plasma of the ionosphere can introduce significant Faraday rotation especially at low radio frequencies during times of enhanced solar activity (Pacholczyk 1970; Cotton 1989). Faraday rotation is a rotation of the apparent orientation of a linearized polarized signal and can be expressed as a time and observing geometry variable right-left phase difference.

The degree of Faraday rotation is a function of the integral of the product of the electron density and the component of the magnetic field parallel to the line of sight. The effect of this integral is that the Faraday rotation will vary strongly with observing geometry even if the ionosphere remains constant. The variable exposure of the ionosphere to photon and charged particle ionization produces strong diurnal variations in the electron density.

Thus, ionospheric Faraday rotation can produce significant right-left phase differences which vary with source at a given time and with time for a given source. This time variable right-left phase difference can severely defocus the polarized image and result in erroneous estimates of the feed polarization parameters.

15.3.7.1 External Faraday Rotation Calibration

An estimate of the Faraday rotation can be obtained from models of the ionosphere and the earth's magnetic field. The magnetic field can be assumed to be constant but the electron density is highly variable. The electron density can be estimated from an empirical model based on solar activity (Chiu 1975) or by one of a variety of direct measurements.

Estimates of the Faraday rotation can be used to correct data in a number of possible ways. The effect of Faraday rotation is similar to that for parallactic angle and can be subtracted from the parallactic angle whenever the parallactic angle is used.

Faraday rotation could be measured from observations of a polarized calibrator but in practice it is usually determined from completely external sources. In this case, a simpler way to correct for Faraday rotation would be to apply these corrections before any calibration determined directly from the interferometric data.

15.3.7.2 Faraday Self-Calibration

It is possible to solve for Faraday rotation corrections in a fashion similar to solving for atmospheric fluctuations in the standard self-calibration procedure, i.e.Faraday rotation self-calibration. If crosshand correlations, after application of parallel-hand and instrumental polarization calibration, are divided by the Fourier transform of a polarization model, the right-left phase difference due to variable ionospheric Faraday rotation can be estimated in each solution interval directly from the phase of the divided RL or conjugate (LR) phases. The total

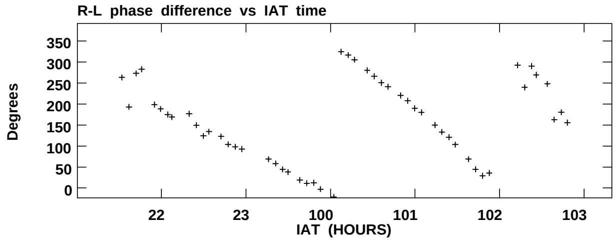

Figure 15.2: This figure shows the difference in the right and left hand phase solutions as a function of time resulting from the Faraday self calibration of a source seriously affected by variable ionospheric Faraday rotation. Over two full turns of right-left phase difference are seen. These observation were made at a frequency of 333 MHz.

intensity calibration should have adjusted the phases of the parallel-hand systems such that the difference between the two systems is described by a single phase difference, that of the reference antenna used for the total intensity calibration. These Faraday rotation corrections can then be applied to the data and a new polarized image derived. Any prior R-L phase offset calibration will be lost in this process. Figure 15.2 shows the time variations of the R-L phase difference determined using this technique for a set of low frequency VLA data.

15.4 Instrumental Polarization Calibration

The instrumental polarization parameters can be determined from observations of a calibrator of known polarization. In practice, sources small enough to be used as a VLBI calibrator are physically small enough that they frequently show time variability in total and polarized intensities as well as structure. Thus, it is frequently the case that the instrumental and calibrator polarizations must be determined jointly.

The effects of variable parallactic angle in observations using antennas with an alt-az mount is different for source and instrumental polarization. The component of the interferometer response due to the source rotates with parallactic angle whereas the response due to the instrumental polarization is constant. These two contributions add vectorially.

The effect of parallactic angle on the instrumental contribution to the polarization of VLBI observations is more than a simple rotation of a constant magnitude vector. The additional effects are easily understood in the context of the ellipticity-orientation model of the feed response. In this model the instrumental polarization is a result of the mismatch of the feed polarization ellipses. Since the parallactic angle is a function of observing geometry it can be very different for antennas at different locations on the earth. More importantly the difference in parallactic angle changes with time, especially near transit at each of the antennas involved. The variable difference in parallactic angle rotates the polarization ellipses with respect to each other causing a variable amplitude

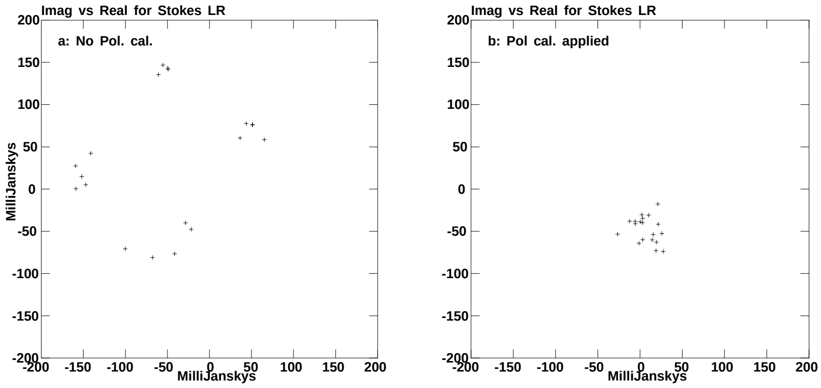

Figure 15.3: (a) This figure shows plots of the real versus the imaginary parts of the RL correlation on a single baseline for a single weakly polarized calibrator observed in four scans. The scatter of the points in each scan indicates the noise. The two antennas involved have alt-az mounts and the phases have been corrected for the effects of parallactic angle. (b) This plot shows the data in (a) after correction for instrumental polarization.

as well as phase of the instrumental polarization. This effect is shown in figure 15.3 which shows polar diagrams of a cross polarized correlation sampled at a number of different parallactic angles. Figure 15.3a shows the data without corrections for instrumental polarization and figure 15.3b shows the same data with instrumental polarization correction applied. The data in figure 15.3 have had parallactic angle corrections applied so the source contribution is not rotated by the parallactic whereas the instrumental polarization is.

The different response with parallactic angle for source and instrumental polarizations allows using observations over a range of parallactic angles to separate these effects and to use a parameter estimation procedure to determine the source and instrumental polarizations. Instrumental polarization corrections should be applied to data after it has been fully amplitude and phase calibrated (self-calibration or phase referencing).

15.4.1 Calibrator Polarization Model

If the polarization calibrator is unresolved its polarization is described simply by its Q and U flux densities. If the source is resolved then its polarized structure must be known. An approximation that can be used to account for resolution of the calibrator is the *similarity* approximation. In this approximation it is assumed that the Q and U images are scaled versions of the I image and only the RR and LL correlations and the fractional Q and U polarization are needed to predict the polarized correlations. The calibration procedure can estimate these fractional polarizations jointly with the instrumental parameters.

For very resolved calibrators this approximation can be quite bad as the polarized images of the calibrator may be quite different from the total intensity images. However, in the limit of an unpolarized source it is a completely accurate model; the fractional Q and U values are zero. As many of the potential polarization calibrators are weakly linearly polarized (a few percent or less) this approximation is a sufficient starting point for the iterative procedure described later. The similarity approximation is also adequate for an unresolved source.

15.4.2 Fitting A Feed Model

The model used for the feed depends to some extent on the quality of the feeds used. For good feeds (few percent instrumental polarization) and modest goals (polarization dynamic ranges of 100:1 or less) then a linearized *D-term* model is preferable as it is easily fitted. If the feeds used are of poor quality, or poorly matched or high polarization dynamic range is desired, then a fully nonlinear model is required.

15.4.3 Iterative Calibration

If the similarity approximation suggested above for a resolved calibrator is inadequate, it is still possible to use the instrumental polarization solution derived for an initial calibration of the calibrator data. This data can be imaged and deconvolved and the resulting polarized model used in a subsequent determination of the instrumental polarization. This procedure should result in improved estimates of the instrumental polarization and can be repeated as necessary. The final instrumental solutions can be used to calibrate the target source data. This scheme could be described as instrumental polarization self-calibration.

15.5 Polarization Angle Calibration

As was discussed in earlier sections, the earth's atmosphere and the instrument can introduce arbitrary phase offsets between the right and left handed systems. Proper calibration techniques can result in a single but unknown phase offset between these two systems for all times and baselines. This phase offset will result in a rotation of the apparent orientation of the E-vectors. The right-left phase difference can only be determined from observations of a source with known polarization angle.

The position angle of the E-vectors of the electric field of the radiation (the orientation of the polarization), is given by:

$$\Phi = \frac{1}{2} \tan^{-1} \frac{U}{Q} \,. \tag{15.22}$$

As $RL = q + iu$, it follows that the calibrated RL phase (or the conjugate of the calibrated LR phase) for a point source at the phase center is twice the apparent angle of the E-vectors. The correction to the right-left phase difference can be determined from the difference in the observed and *true* right-left phase differences.

15.5.1 Unresolved Calibrator

A correction to the right-left phase difference can be determined from direct inspection of the calibrated RL (or LR^*) phases of a point source of known polarization. Since the right-left phase difference is clearly related to the phases of the parallel-hand systems it is desirable to incorporate the right-left difference into the parallel-hand calibration. Since the polarization parameters were determined without application of the right-left phase difference, they will have to be suitably adjusted if the right-left phase difference is incorporated into the parallel-hand calibration. The details of this adjustment depend strongly on the model of the feed polarization.

The right-left phase difference correction can alternately be applied as a correction to the position angles derived from the Q and U images. This requires no modification of the parallel-hand phase calibration or of the feed polarization calibration parameters.

15.5.2 Resolved Calibrator

If the calibrator source is resolved (or not at the phase center) then the observed RL and LR phases will not be simply related to the right-left phase difference. If the source is substantially unresolved on a subset of the baselines then this subset can be used to estimate the right-left phase difference as in the completely unresolved calibrator case.

In many cases the calibrator source is resolved on all baselines, or the signal-to-noise ratio or polarization calibration quality on baselines for which it is unresolved is inadequate. In these cases, the total source polarization parameters can be estimated by integrating the Q and U flux in the image of the calibrator (sums of the CLEAN component fluxes are equivalent). The apparent, integrated polarization angle is given by:

$$\Phi = \frac{1}{2}\tan^{-1}\frac{\Sigma U}{\Sigma Q}\,. \qquad (15.23)$$

The correction to the right-left phase difference can be determined and applied as in the unresolved calibrator case.

15.6 Imaging

The techniques used for imaging and deconvolving VLBI data are covered in detail in other lectures although some adaptations to the imaging process are necessary for polarized images. The usual technique is to convert the measured RL and LR correlations to q and u correlations and make and deconvolve Q and U images independently. This is adequate if there is sufficient (u, v) coverage using data with both RL and LR.

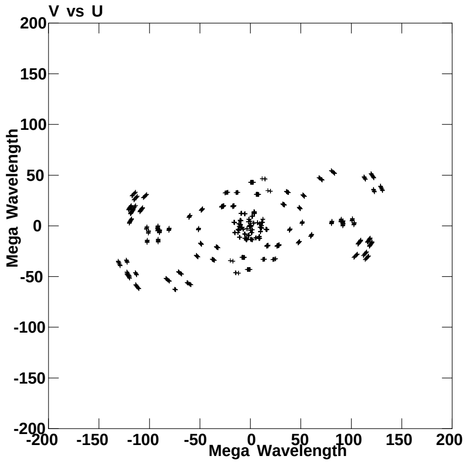

Figure 15.4: The polarization (u, v) coverage of observations of a source with the RL correlations plotted in one half plane and LR in the other. One of 9 antennas recorded a single polarization and there was some editing of one polarization for the other antennas.

15.6.1 Asymmetric (u, v) Coverage

If it is necessary to use data with single cross-polarized measurements (RL or LR but not both) it is possible to make a complex $Q + jU$ image (Conway and Kronberg 1969). The RL and LR correlations sample conjugate points in the $q + ju$ function. As the $Q + jU$ image is complex, its Fourier transform is asymmetric so the RL and LR correlations are independent measures of the source. Figure 15.4 shows the polarized (u, v) coverage from observations of a source with mildly asymmetric coverage.

15.6.2 Complex Deconvolution

An asymmetric sampling function caused by missing RL or LR correlations will Fourier transform to a complex dirty beam. Thus, the derived complex $Q + jU$ image will be the convolution of the sky $Q + jU$ with this complex dirty beam plus noise. A complex deconvolution will produce a Q image as it real part and a U image as its imaginary part. An adaptation of both the CLEAN and MEM deconvolutions are discussed in Cotton et al. (1984). An example of a complex dirty polarization beam in shown in figure 15.5.

15.7 Spectro-Polarimetry

Most of the discussion of continuum polarimetry is directly applicable to spectroscopy data although there are several, possibly serious, complications. The most serious complication is that the assumption of no circular polarization can

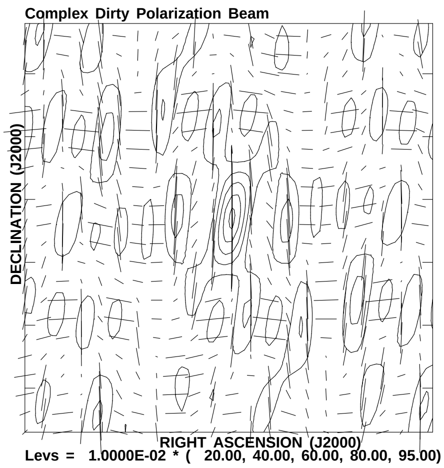

Figure 15.5: Shows the inner portion of a dirty polarization beam. Contours show the amplitude and the orientation of the vectors give the angle in $Q-U$ space. The (u,v) coverage is that shown in figure 15.4. The coverage is nearly symmetric so most of the power in the beam is real (vertical vectors).

be very wrong. OH masers in particular may be highly circularly polarized.

Spectroscopic observations can have instrumental polarization determined from a continuum calibrator so the main complications are in the phase calibration of the spectroscopic data. The atmospheric phase fluctuations can be estimated from one of the parallel polarizations and applied to the other; fluctuations due to the neutral atmosphere should be the same for the two polarizations. In determining the instrumental contributions for spectroscopic data, the assumption that RR is equivalent to LL should be avoided.

Another complication with spectroscopic polarization measurements is that the bandpasses may be quite narrow making calibrations using continuum source more difficult. The intrinsically narrow frequency width of spectral features means that fringe fitting of sources with no continuum emission can determine only phase and rate residuals. Continuum calibration sources must be used for delay calibration.

Instrumental phase cals are generally not used in spectroscopic observations for a variety of reasons. This makes maintaining a constant right-left phase difference at each antenna more difficult. This increases the requirement for inherent stability in the antenna cabling and electronics especially at the reference antenna. Frequent observations of a continuum calibrator may be required to achieve right-left instrumental stability. Kemball, Diamond and Cotton (1995) describe the process of calibration of spectro-polarimetric VLBI data in more detail.

15.8 Suggested Procedure

This section outlines a general procedure for calibrating a set of continuum VLBI polarization measurements. Details of particular programs (or even analysis packages) will not be given as these are subject to rapid evolution. This procedure assumes that all calibration is done on an antenna basis and that the data are kept in *raw* form and a cumulative set of calibration values determined and applied as necessary.

1. Set antenna mount types

   The antenna information with the data set should correctly specify which antennas have alt-az and which have equatorial mounts.

2. Parallactic angle correction.

   Remove the effects of parallactic angle from the antenna phases.

3. Amplitude calibration.

   Use measured or estimated system temperature values and antenna sensitivity estimates to calibrate the amplitudes and weights.

4. Delay and phase adjustment.

   A short time segment of calibrator data should be used to correct phase cals and align single- and multi-band delays. No fringe rate corrections should be applied at this point. This adjustment also adjusts the right-left phase differences for all antenna to that of the reference antenna. This technique is discussed in more detail in the lecture on fringe fitting.

5. Fringe fitting.

   The data in the two parallel systems should be fringe fitted independently. See the lecture on fringe fitting for more details. The fitted values should be filtered and referred to a common reference antenna to maintain the phase and delay relationship between the right and left handed systems and applied to the calibration.

6. Check Right-left coherence.

   Make and examine plots of the difference in the right and left hand filtered phase solutions from the fringe fit as a function of time for each antenna (see fig. 15.1). Step 4 should have adjusted the right-left phase differences for all antennas to be near zero. The values should all show a small scatter about a common value. Differences between antennas or abrupt changes in the difference for a given antenna are indicators of problems referring the phase to a common reference antenna or other problems affecting the antenna instrumental right-left phase difference. Right-left phase differences that vary smoothly with time or with source on all antennas are indicators of ionospheric Faraday rotation. Variations on only a few antennas may indicate instrumental rather that Faraday rotation problems. Any problems found at this stage should be fixed before proceeding further.

7. Right-left delay differences.

   A short time segment of cross polarized data can, after application of the parallel-hand calibration, be used to determine the right-left single- and multi-band delay differences. One or more baselines with good signal-to-noise ratio can be used in this determination; a single set of these delay differences should be applicable to all baselines and times. These corrections should then be applied to one of the parallel systems so that the application of the parallel calibration to the cross polarized data will remove the residual delays and rates. Note, a correction made to all antennas in a single polarization will not affect the parallel-hand calibration as each baseline correction is the difference of the antenna values.

8. Self-calibration of calibrators.

   The data for each calibrator should be self-calibrated to produce a total intensity image. The solutions should be referred to a common reference antenna to avoid disrupting the phase relationship between the right and left handed systems.

9. Instrumental polarization.

   Use the fully amplitude and phase calibrated calibrator data to jointly determine the calibrator and feed polarizations. If the feeds have relatively pure polarization then a linearized model may be used; otherwise a non-linear model such as the ellipticity-orientation model should be used. The similarity approximation for resolved calibrators may be necessary. For significantly polarized, resolved calibrators this approximation may not be adequate. In this case the similarity approximation can be used anyway and the derived feed polarizations used to calibrate the calibrator data. This data can then be imaged and deconvolved and the resulting model used in a subsequent determination of the feed parameters.

10. Calibration of the polarization orientation.

    A correction to the right-left phase difference can be determined from observations of a source with known polarization orientation. The details depend on the resolution of the calibrator. The calibration phases (and instrumental polarization parameters) should be adjusted.

11. Calibration of target source data.

    The target source should be fully amplitude and phase calibrated using either self-calibration or phase referencing or a mixture of techniques. Corrections for instrumental polarization and polarization angle should then be applied.

12. Imaging.

    If there is adequate (u, v) coverage using only data with both RL and LR correlations then these can be converted into q and u correlations which can then be used to produce Q and U images. If use of asymmetric coverage is necessary then a fully complex imaging and deconvolution technique should be applied.

References

Brown, L. F., Roberts, D. H., & Wardle, J. F. C. 1989. "Global fringe fitting for polarization VLBI". *Astron. J.* **87**, 1522.

Chiu, Y. T. 1975. "An improved phenomenological model of ionospheric density". *J. Atm. and Terr. Phys.* **37**, 1563.

Clark, B. G. 1989. "Coherence in radio astronomy". In R. A. Perley, F. R. Schwab, and A. H. Bridle (Eds.), *Synthesis Imaging in Radio Astronomy*. San Francisco: Astronomical Society of the Pacific, p. 1.

Conway, R. G., & Kronberg, P. P. 1969. "Interferometric measurements of polarization distributions in radio sources". *Monthly Notices Roy. Astron. Soc.* **142**, 11.

Cornwell, T. J., & Perley, R. A. (Eds.) 1991. *Radio Interferometry: Theory, Techniques, and Applications*, Volume 19 of *ASP Conference Series*. San Francisco: Astron. Soc. of the Pacific.

Cotton, W. D. 1989. "Polarimetry". In M. Felli and R. Spencer (Eds.), *Very Long Baseline Interferometry*. Volume 283 of *NATO ASI Series C*. Dordrecht: Kluwer, p. 275.

Cotton, W. D. 1993. "Calibration and imaging of polarization sensitive VLBI observations". *Astron. J.* **106**, 1241.

Cotton, W. D., Geldzahler, B. J., Marcaide, J. M., Shapiro, I. I., Sanroma, M., & Rius, A. 1984. "VLBI observations of the polarized radio emission from the quasar 3C454.3". *Astrophys. J.* **286**, 503.

Fomalont, E. B., & Wright, M. C. H. 1974. "Interferometry and aperture synthesis". In G. L. Verschuur and K. I. Kellermann (Eds.), *Galactic and Extragalactic Radio Astronomy*. Berlin: Springer, p. 256.

Kemball, A. J., Diamond, P. J., & Cotton, W. D. 1995. "Data reduction techniques for spectral line polarization VLBI observations". *Astron. Astrophys. Suppl.* (110), 384.

Leppänen, K. J., Zensus, J. A., & Diamond, P. D. 1995. "Linear polarization imaging with very long baseline interferometry at high frequencies". *Astron. J.*. in press.

Lüdke, E. 1993. "A new polarization calibration technique for a large phased array with a maximum baseline 220 km". *IEEE Int. Conf. Ant. Prop.* 1(8), 610.

Noordam, J. E. 1991. "High acuracy ($<< 1\%$) polarization with the WSRT". See Cornwell and Perley (1991), p. 206.

Pacholczyk, A. G. 1970. *Radio Astrophysics*. San Francisco: Freeman.

Roberts, D. H., Brown, L. F., & Wardle, J. F. C. 1991. "Linear polarization sensitive VLBI". See Cornwell and Perley (1991), p. 281.