Simulations of the Effects of 1/f Gain Fluctuations on Measuring Linear Polarization with Linear Feeds on ALMA

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Abstract

Through polarization imaging simulations in AIPS++ which do not include instrumental polarization leakage or phase errors, we determine that 1/f gain fluctuations of magnitude 1e-3 in 300 s will not prevent us from obtaining the fractional polarization specification of 0.001 for images of intermediate complexity. In very complicated objects, we expect there will be a limitation on the fractional polarization of weaker pixels.

1 Simple POL Background

Anytime we difference two large numbers to try to estimate the magnitude of their small difference, astronomers should be worried. Thats the situation researchers who study Zeeman splitting with the VLA find themselves, as they must estimate the magnitude of Stokes V by differencing the RR and LL visibilities or the RR and LL images. Small gain differences in the RR and LL systems can result in spurious polarization signals, limiting our ability to see polarized astronomical signals with small fractional polarization.

ALMA, with linear feeds, has a similar difficulty in measuring linear polarization. The linearized polarization leakage equations from Cotton (MMA Memo 208) shows, to second order, what signals exist in the XX, YY, XY, and YX correlations:

$$XX = g_{1x} g_{2x} (I + Q \sin(2\chi) + U \cos(2\chi))$$

$$YY = g_{1y} g_{2y} (I - Q \sin(2\chi) - U \cos(2\chi)),$$

$$XY = g_{1x} g_{2y} ((d_{1x} - d_{2y}*) I - Q \sin(2\chi) + U \cos(2\chi)),$$

$$YX = g_{1y} g_{2x} ((-d_{1y} + d_{2x}*) I - Q \sin(2\chi) + U \cos(2\chi)).$$

AIPS++ doesn't deal with these linearized equations, but this is a simple place to look to see what is going on with the signals. In the above equations, I is actually the Fourier Transform of the I image evaluated at the $1, 2$ baseline, and Q and U are the Fourier transforms of those Stokes images, evaluated at the measured baseline.

The parallel correlations are Stokes I modulated by Q and U bobbing up and down with the sin and cos of twice the parallactic angle, and the difference of XX and YY, measured

Time Scale $[s]$	Fractional RMS
10	0.00055
30	0.00072
100	0.00087
300	0.00097

Table 1: RMS difference in gain errors vs time lag. Unlike the Allan Standard Deviation, the time series is merely sampled and differenced. The Allan Standard Deviation is calculated by averaging the gain time series before calculating the RMS.

over a long strentch of parallactic angle, provides us with a means of extracting Q and U. However, this estimate will be contaminated by gain amplitude fluctuations due to 1/f noise. The receiver and IF systems are required to have no more than 5e-4 rms gain fluctuations over 300 s. Will gain errors of this magnitude prevent ALMA from reaching its imaging specification of reaching 0.001 in fractional polarization?

There is another source of polarization information: the cross hand correlations XY and YX also have information on Stokes Q and U, but these correlations will not be so stringly affected by the gain fluctuations, as they don't have the full strength of Stokes I in them (only through the instrumental polarization leakage).

In AIPS++, the parallel and cross hand visibilities are converted into Stokes components and imaged: both the XX-YY difference and the XY, YX visibilities are used to estimate the Stokes parameters, so the largish XX-YY error gets injected into the polarization images.

In the event that the parallactic angle coverage is very good, one can rely mainly upon the XY and YX correlations plus the parallactic angle variable to disentangle the Q and U signals. One imagine a different polarization algorithm to apply when we are limited by gain fluctuations which looks at the parallactic angle coverage to determine with what weight we need to include the XX-YY difference in the polarization imaging.

2 Simulation Setup

We simulated a 1 hour observation with 1/f gain errors and thermal noise. The gain errors are defined as having a flat Allan Standard Deviation of 5e-4. This actually results in somewhat larger rms fluctuations for longer time scales, as is represented in Table 1 (notably, the rms fractional difference of two gains 300 s apart in these simulations were almost 1e-3). Additionally, it is assumed that the gain errors can be accurately determined through calibration every 1200 s.

Two different model brightness distributions were used: a 1 Jy point source, and a source which was made up of 25 Jy spread among 9 Gaussians, which made for an extended source of of about 15x15 beams. A point source model should be the easiest thing to image, as all the visibilities are adding up to tell us about that one pixel in the center of the image, and there will be a lot of averaging. Of course, the flucuating gains will also spuriously scatter total intensity flux into the polarization images, obscuring the detection of real polarized emission in weak extended features which might surround a point source, but we neglect this aspect of the problem at this time. The more extended model brightness distribution (without the point source) will result in less averaging of visibilities and larger errors in fractional polarization. In both cases, the model source was unpolarized, so any signal in the polarization image will be due to thermal noise or 1/f gain errors. (Evaluating the polarization errors on a polarized test case with a polarized image supported this assertion.)

3 Simulations Results

The results for the point source simulations are shown in Figure 1, which shows the RMS error in stokes Q and P images (the error in linear polarization will actually be sqrt(2) higher) as a function of the thermal noise level per visibility per 10 s integration. For high thermal noise levels, the polarization image RMS is linear with the noise (ie, the imaging is thermal noise limited). However, as the noise level drops below 0.01 Jy (ie, the SNR on a single 10 s visibility for this 1 Jy model source exceeds 100), the RMS in the polarization image flattens out and we become dominated by the gain fluctuations. Apparently, for the simple case of a point source, the 5e-4 gain fluctuations do not limit detection of linear polarization at the level of better than 1e-5 in fractional polarization, greatly exceeding the 1e-3 fractional polarization specification.

Figure 2 shows the analagous plot for the extended source. The peak brightness in the extended source is lower than the point source, so a given noise level will represent a lower SNR, and will result in a higher RMS fractional polarization error. We see that the RMS polarization error decreases linearly with the RMS noise level, and for these simulations we do not go down low enough in noise level, as we don't actually see the onset of being limited by the 1/f gain fluctuations at low noise levels. For this model source, we do better than 1e-4 in fractional polarization averaged over the source.

4 Warning About the Fractional Polarization Specification

Note that the specification on the fractional polarization level is ill-conceived. As we have shown, on a bright point source we have no problem achieving the 0.001 fractional polarization level. However, it is not reasonable to assume that we can achieve a fraction polarization of 0.001 anywhere in the image when the image is dominated by a bright point source, as errors will be scattered from the bright point source and may limit the polarization on weak features off of the point source.

5 Total Power Observations

The effect of 1/f gain fluctuations on total power imaging may be more severe than on interferometric imaging, as there will often be only four total power antennas, and the X and Y gain errors will be more correlated due to the receivers being collocated, which could make the problem worse than the interferometric case by sqrt(2). We have not performed any simulations of total power imaging. However, we can reason that in the event that we are noise dominated in the total power imaging on time scales of 1200 s (ie, if we had noise which was greater that one part in 10^3 on the time scale over which we are assuming we can periodically measure the gains fairly accuratly), then we will not be limited by the 1/f gain fluctuations:

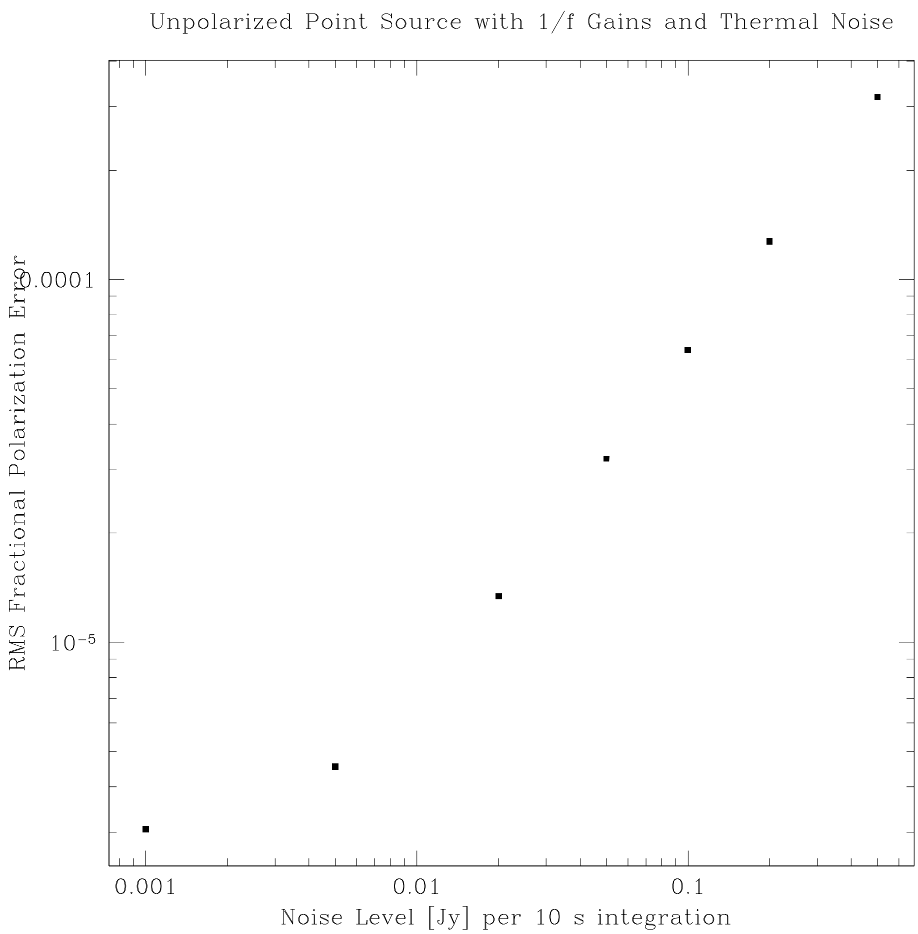

Figure 1: Interaction with thermal noise and 1/f gain errors for a 1 Jy point source: as the thermal noise per 10 s visibility decreases (with the same 1/f gain fluctuations), the image plane polarization error decreases also (ie, follows the noise), but eventually at very low noise, the image plane polarization error is limited by the 1/f gain fluctuations.

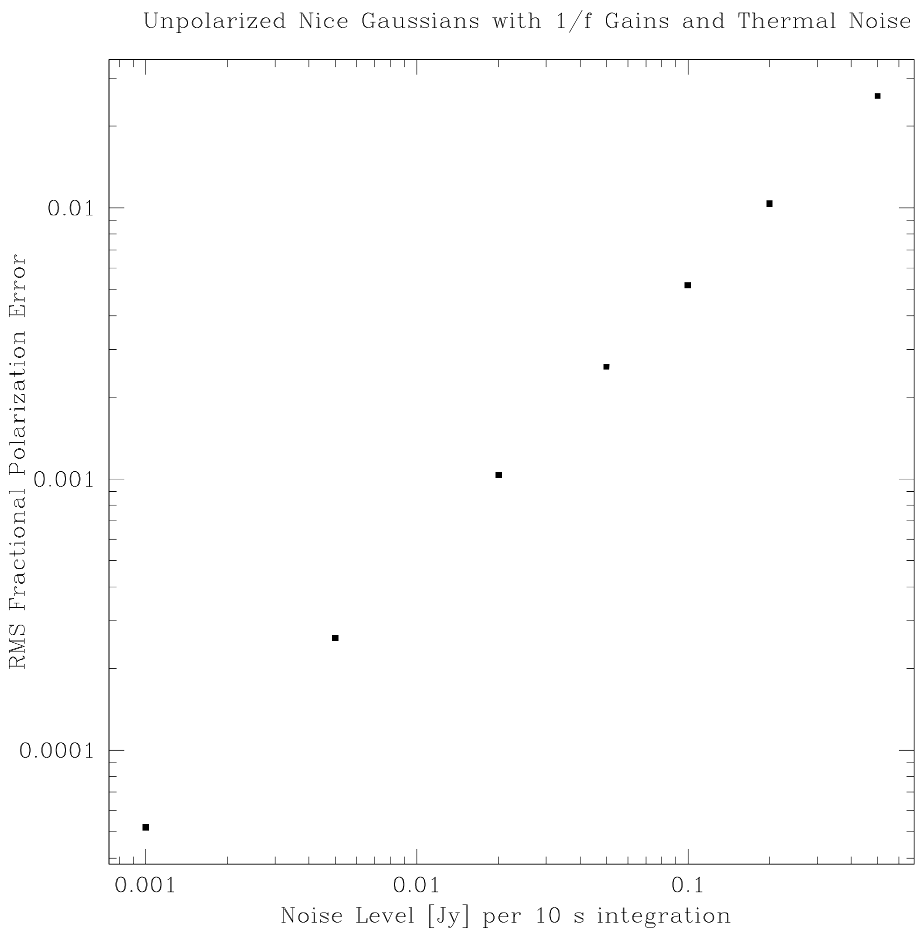

Figure 2: Interaction with thermal noise and 1/f gain errors for a 25 Jy extended source: as the thermal noise per 10 s visibility decreases (with the same 1/f gain fluctuations), the image plane polarization error decreases also (ie, follows the noise).

long observations should average down. For the brightest sources, this will not be true, and we may be limited by the 1/f gain fluctuations.

6 Provisional Conclusions

UPDATE THESE CONCLUSIONS

Polarization imaging imulations in AIPS++ including 1/f gain fluctuations indicate 1/f gain fluctuations at the level of 1e-3 over 300 s will not limit the detection of linear polarization at the 0.001 level (in fractional polarization) in most cases.

We have not included "d" terms, antenna-based phase errors, or X-Y phase offsets in these simulations.

What about other errors which might limit the polarization images? Errors in the d terms translate, to first order, into leakage from the total intensity image into the polarization image, approximately as the mean of the combination of residual d terms (whats left of them after calibration): $d_{x,i} + d^*_{y,j}$ (averaged over all baselines). At the VLA, I found the polarization imaging was limited due to fluctuations in the d terms on timescales ranging from minutes to days. At the ATF, which has linear feeds, the experience has been that the d terms are very stable for months at a time (the argument is that it is just geometry, while the VLA's circular feed d terms have standing waves and electronics as well contributing), so we hope that the ALMA will have very stable d terms which we can solve for accurately. If so, then the ALMA polarization will probably be able to go down to 0.001 in fractional polarization or lower.

Errors in the phases of the gains due to the atmosphere could limit the fractional polarization for snapshots of large complicated objects, but that isn't a very realistic case: if you use a snapshot, you probably have a less complicated object, and if you have a complicated object, you'll probably have longer tracks with more averaging. Fast switching will typically have 20 deg phase errors on each visibility, but the fast switching process, performed about once every 30 s, will randomize those errors pretty well, and they will average down assuming there is enough time, or there are enough different baselines contributing to a particular (u,v) cell and the number of image plane pixels we are solving for is not overwhelming. The X-Y phase offsets need to be studied more.

Cotton, W.D., "Polarization Calibration of the MMA: Circular vs Linear Feeds", MMA Memo 208, 1998.