A New Method for Cross Polarized Delay Calibration of Radio Interferometers

W. D. Cotton, January 8, 2012

Abstract—**The usual technique for independent calibration of parallel–hand visibilities, RR & LL or XX & YY allows for an arbitrary offset in delay and phase between the two parallel systems. In order to use the cross–polarized visibilities, this offset must be determined and removed. This memo describes one such technique and explores its application in the Obit package. The technique is successfully applied to some EVLA data using both strongly and weakly polarized calibrators.**

Index Terms—**interferometry, polarization, calibration**

I. INTRODUCTION

RAdio interferometry provides a powerful tool for probing the structure of the polarised emission from celestial sources. To utilize such observations, corrections must be applied for the effects of the independent electronics, paths through the atmosphere and errors in the assumed geometric model that corrupt the data.

The traditional calibration technique for radio interferometric data, especially when the receptors measure the orthogonal circular polarizations, has been to determine the calibration for the two hands of polarization separately. This effectively ignores both the polarization of the calibrator and the spurious polarized response of the instrument.

For systems measuring circular polarization and using synchrotron sources for calibration, this is generally a good approximation. The circular polarization of the typical compact radio source is less than 0.1% and the spurious instrumental polarization response of the order of a few %. This approximation is less valid for interferometers measuring orthogonal linearly polarized signals as the same calibrators typically have linear polarizations of a few to ~10%.

Independent calibration of the parallel–hands also has the disadvantage that it allows an arbitrary delay and phase offset between the two parallel systems. The cross–polarized visibilities may be dominated by the linear polarization of the source which can be used for cross–polarized calibration. The offsets between the parallel–hands need to be determined and removed before the cross–polarized visibilities can he used for astronomical imaging.

One technique used in the past was to "fringe fit", fit for delay and delay rate, independently for the parallel–hand visibilities[1], apply this calibration and then do single baseline delay fitting of a a few cross–polarized visibility spectra and average the results [2]. An alternate technique [3] is to use closure constraints on the results of a series of single baseline fringe fits to determine the offsets between the parallel hand

National Radio Astronomy Observatory, 520 Edgemont Rd., Charlottesville, VA, 22903 USA email: bcotton@nrao.edu

systems. Both of these techniques have limited sensitivity. This memo explores an improved technique in the Obit package [4] [1].

II. INTERFEROMETRIC POLARIMETRY

Telescopes using heterodyne electronics as are commonly used in interferometers at radio wavelengths are sensitive to a single polarization state of the incoming wave. In order to fully sample the celestial signals, sets of electronics nominally sensitive to orthogonal polarizations are used. To sample the full state of the visibility measured with an interferometer, cross–correlations of all (4) combinations of the polarizations states are made on each baseline, or pair of antennas. Denote the various visibilities measured between antennas j and k and using detectors for polarization states p and q as v_{jk}^{pq}. Typically, either right and left–hand circular polarizations ("R", "L") or orthogonal linear polarizations ("X", "Y") are used in radio interoferometers.

In practice, the two polarizations measured are not precisely those desired but can be modeled by the desired state plus a complex value, called the "leakage" term, d_{jp} times the orthogonal state. Thus, the signal received by a detector on antenna j nomimally in polarization state p is actually:

$$s'_{jp} = s_{jp} + d_{jp}s_{jq} \qquad (1)$$

To first order, the interferometric response for an interferometer between antennas j and k using linear detectors of an unresolved source is [5]:

$$
\begin{aligned}
v_{XX} =& \frac{1}{2}g_{jX}g_{kX}^*(I + Q\cos2\chi + U\sin2\chi, \\
v_{XY} =& \frac{1}{2}g_{jX}g_{kY}^*[(d_{jX} + d_{kY}^*)I - Q\sin2\chi + U\cos2\chi + iV)], \\
v_{YX} =& \frac{1}{2}g_{jY}g_{kX}^*[(d_{YX} + d_{kX}^*)I - Q\sin2\chi + U\cos2\chi - iV)], \\
v_{YY} =& \frac{1}{2}g_{jY}g_{kY}^*(I - Q\cos2\chi - U\sin2\chi),
\end{aligned}
$$

$$(2)$$

where g_{jp} is the complex gain of the electronics for polarization p on antenna j, "*" denotes the complex conjugate, I, Q, U, and V are the Stokes parameters of the source emission, i is $\sqrt{-1}$ and, χ is the parallactic angle given by:

$$
\chi = \tan^{-1}\Big(\frac{\cos\lambda\sin h}{\sin\lambda\cos\delta - \cos\lambda\sin\delta\cos h}\Big) \qquad (3)
$$

where δ is the source declination, λ is the latitude of the antenna and h is the hour angle of the source. For linearly

polarized detectors with detectors rotated from the local horizontal and vertical, this rotation needs to be added to the value of χ given above. To simplify the notation in the following, we assume nearly identical parallactic angles at all antennas at a given time.

The corresponding relations for antennas with circular detectors is:

$$
\begin{aligned}
v_{RR} &= \frac{1}{2}g_{jR}g_{kR}^*(I + V), \\
v_{RL} &= \frac{1}{2}g_{jR}g_{kL}^*[(d_{jR} + d_{kL}^*)I + e^{-2i\chi}(Q + iU)], \\
v_{LR} &= \frac{1}{2}g_{jL}g_{kR}^*[(d_{jL} + d_{kR}^*)I + e^{-2i\chi}(Q - iU)], \\
v_{LL} &= \frac{1}{2}g_{jL}g_{kL}^*(I - V).
\end{aligned} \tag{4}
$$

For more details on the response of an interferometer to a partially polarized signal see [5].

III. INDEPENDENT PARALLEL HAND CALIBRATION

If the calibrator and instrumental polarizations are sufficiently small to be ignored, the two parallel polarized sets of visibilities can be assumed to be independent measurements of the same celestial quantities. For a given baseline, $j - k$, the measured correlations ($v_{jk}^{pp\_obs}$) are related to the calibrated visibilities ($v_{jk}^{pp\_cal}$) by:

$$
v_{jk}^{pp\_cal} = v_{jk}^{pp\_obs} \, g_j^p \, g_k^{p*} \tag{5}
$$

where g_j^p is given by:

$$
g_j = a_j^p \, e^{-2\pi i(\Delta\tau_j^p\nu)-\phi_j^p},
$$

and a_j^p is the amplitude correction, $\Delta\tau_j^p$ the delay residual from the correlator model, ϕ_j^p the model phase residual, and ν the observing frequency. Calibration parameters a_j^p, $\Delta\tau_j^p$ and $\phi_j^p p$ can be determined from observations of (calibrator) sources of known brightness, structure and position. Calibration quantities are generally a function of time.

This system of equations is degenerate in that only differences in $\Delta\tau_j^p$ and ϕ_j^p are actually measured. The system is generally made determinate by assigning the values of a "reference antenna" to zero. Thus, all phase–like quantities are determined relative to the reference antenna leaving only the relationship between the two hands of polarization at the reference antenna undetermined.

IV. RESIDUAL CROSS–HAND DELAY/PHASE OFFSET

Independent calibration of the parallel–hand systems allows for a difference between $\Delta\tau_r^p$ and $\Delta\tau_r^q$ as well as between ϕ_r^p and ϕ_r^q where r denotes the reference antenna. Denote these differences as $\Delta\tau_r^{pq}$ and ϕ_r^{pq}. Note: if the observing bandwidth is divided into multiple "spectral windows" with at least partially independent electronics and signal paths, the calibration parameters may vary between sub-windows. The relation per spectral window between parallel–hand calibrated data ($v_{jk}^{pq\_cal}$) and cross–hand calibrated data ($v_{jk}^{pq\_xcal}$) is:

$$
v_{jk}^{pq\_xcal} = v_{jk}^{pq\_cal}e^{-2\pi i\Delta\tau_r^{pq}\nu-\phi_r^{pq}}. \tag{6}
$$

Note: there is a single pair of values per spectral window, $\Delta\tau_r^{pq}$ and ϕ_r^{pq} needed to calibrate the cross–polarized visibilities. A sample baseline of EVLA (circular polarization) calibrator data with parallel–hand but not cross–hand calibration applied is shown in Figure 1.

V. A METHOD OF DETERMINING THE CROSS–HAND OFFSET

While there are but two parameters to be determined per spectral window, there are a number of complications. The most serious of these is spurious instrumental polarization (d_{jp} in Eqs. 2 and 4) which can contribute a significant fraction of the strength of the source polarization to the cross–polarized visibilities. This spurious instrumental polarization is independent of source polarization and can vary strongly with frequency and baseline.

A second complication is for arrays with alt-az mounts for which the antenna rotates with parallactic angle as seem by the source (Eq. 3). If no correction is made and the antennas detect circular polarization, a varying parallactic angle will cause the source polarization to rotate in polarization angle. If the data are corrected for parallactic angle, the instrumental polarization then rotates while the source polarization is constant.

A further complication is the case in which the source has significantly resolved polarized structure. In this case, the source component of the cross–polarized response will vary with time, frequency and baseline.

A. Circularly Polarized Detectorss

Consider the case of the EVLA with alt-az mounts and detectors sensitive to circular polarization observing an unresolved source whose linear polarization is much stronger than the instrumental polarization. If this data has the phases corrected for parallactic angle and has parallel–hand calibration applied, the cross–polarized spectra are from Eq. 4:

$$
\begin{aligned}
v_{jk}^{RL\_cal} &\approx (Q + iU)e^{+2\pi i(\Delta\tau_r^{pq}\nu)+\phi_r^{pq}} \\
v_{jk}^{LR\_cal} &\approx (Q - iU)e^{-2\pi i(\Delta\tau_r^{pq}\nu)-\phi_r^{pq}}
\end{aligned} \tag{7}
$$

Thus, the RL and LR spectra are the complex conjugates of each other and this relationship holds for all baselines and times. In the case represented here, the cross–polarized spectra can be averaged over all baselines and times and the averaged spectra used to fit for $\Delta\tau_r^{pq}$ and ϕ_r^{pq}.

The solution for $\Delta\tau_r^{pq}$ and ϕ_r^{pq} can be separated and done independently. The delay ($\Delta\tau_r^{pq}$) can be determined from the value of $\Delta\tau$ which maximizes:

$$
|\Sigma(v_{jk}^{RL\_cal} + v_{jk}^{LR\_cal*})e^{2\pi i(\Delta\tau\nu)}| \tag{8}
$$

using a direct parameter search over $\Delta\tau$ and $||$ denotes the modulus. The summation is over the spectra.

The phase (ϕ_r^{pq}) is then the phase of the average of the delay corrected visibilities:

$$
\phi_r^{pq} = arg[\Sigma(v_{jk}^{RL\_cal} + v_{jk}^{LR\_cal*})e^{2\pi i(\Delta\tau_r^{pq}\nu)}] \tag{9}
$$

The "signal–to–noise ratio" (SNR) of the fit can then be estimated from the RMS scatter of the phases in radians of
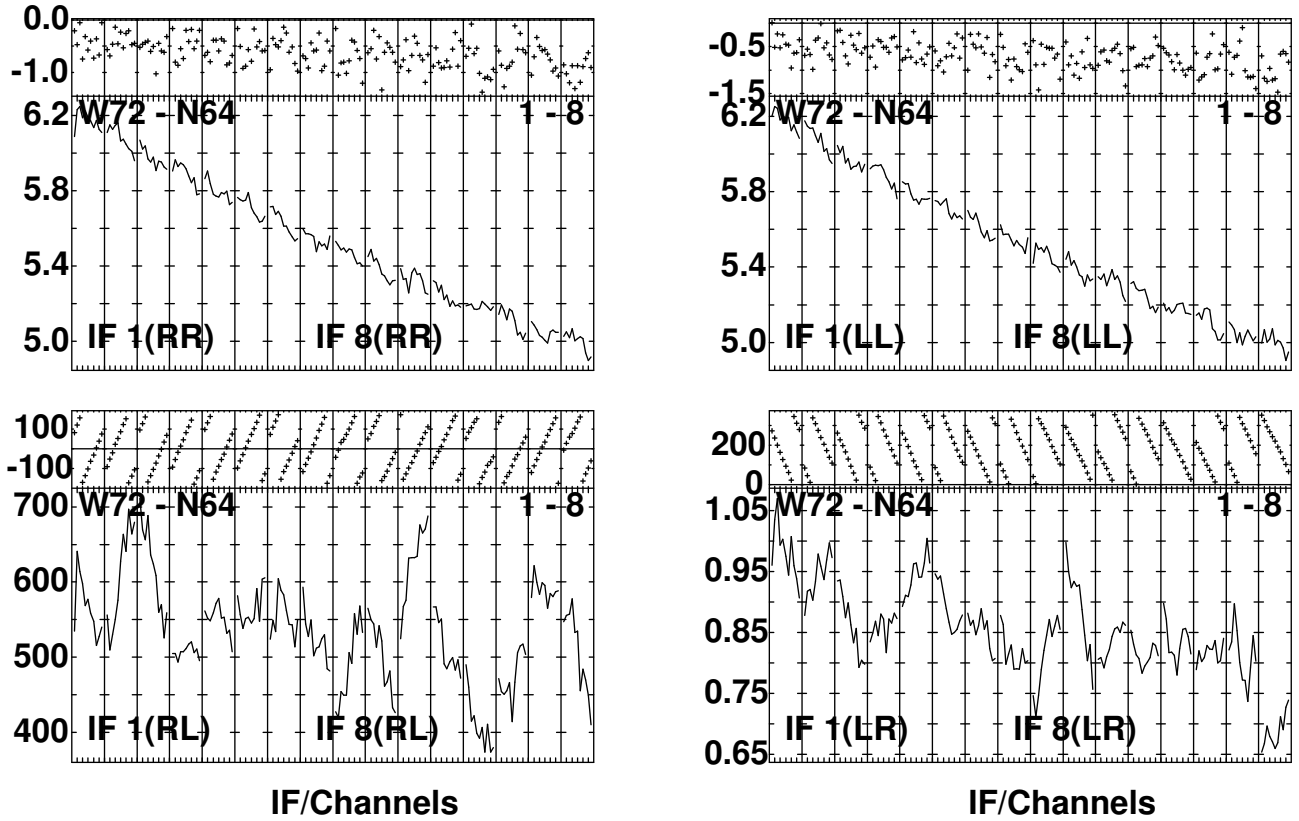
Fig. 1. Sample EVLA visibility spectra for 3C286 averaged over a 15 second scan. Parallel hand calibration has been applied but not the differences between the R and L systems. The upper plot in each panel gives the phase and the lower, the amplitude. Note the rapid variation of cross–polarized phase with frequency indicating a large delay offset.

the individual spectral channel phases about the mean and the approximation that:

$$SNR \approx \frac{1.0}{RMS_{phase}} \qquad (10)$$

In general, the instrumental polarization is not negligible compared to the source polarization but should vary from baseline-to-baseline. Furthermore, after the data are corrected for the parallactic angle, the instrumental polatization will vary with parallactic angle, hence time. Averaging over baselines and time will reduce the contribution of the instrumental polarization to the averaged RL and LR spectra.

B. Linearly Polarized Detectors

The case of interferometers, such as ALMA or the ATCA, whose elements are sensitive to linear polarization is more complex. As can be see from Eq. 2, the source polarization component of the cross–hand visibility spectrum is:

$$
\begin{aligned}
v_{jk}^{XY\_cal} &\approx (Q\sin2\chi + U\cos2\chi + iV)e^{+2\pi i(\Delta\tau_r^{pq}\nu)+\phi_r^{pq}} \\
v_{jk}^{YX\_cal} &\approx (Q\sin2\chi + U\cos2\chi - iV)e^{-2\pi i(\Delta\tau_r^{pq}\nu)-\phi_r^{pq}}.
\end{aligned}
\qquad (11)
$$

Stokes V for most sources is very small and can be ignored. However, the linearly polarized component is a more complex function of parallactic angle and can only be averaged over a time range and set of baselines for which the variation of parallactic angle is small.

The averaged XY and YX spectra can be used to determine the X-Y delay difference as was done for circular feeds in the section above and the SNR of the fit can be determined as well. Variation of the instrumental polarization among baselines will reduce its effect on the averaged spectra.

VI. RLDly: An Obit Implementation

The technique outlined above has been implemented in the Obit[4] package with the fitting done in the utility module ObitUVRLDelay which is available through task RLDly. The fitting is that described in the previous sections except using weighted sums to fit for $\Delta\tau_r^{pq}$ and ϕ_r^{pq}. The implementation in RLDly allows for dropping the end channels in each spectral window which may be poor representations of the values being fitted. In this case the phase ramp applied to the spectra still begins with channel 1.

Data may also be selected by source, antenna, time range, and UV range. Target values of the cross–polarized phase difference and rotation measure may be specified. After fitting $\Delta\tau_r^{pq}$ and ϕ_r^{pq} in each spectral window, an AIPS SN table is written which can be applied to an AIPS CL table using Obit task CLCal. The fitted quantities are written as the second polarization in each AIPS SN table record:

```
row->Real2[i]   =   cos(phase[i]);
row->Imag2[i]   =  -sin(phase[i]);
row->Delay2[i]  =   delay[i];
```

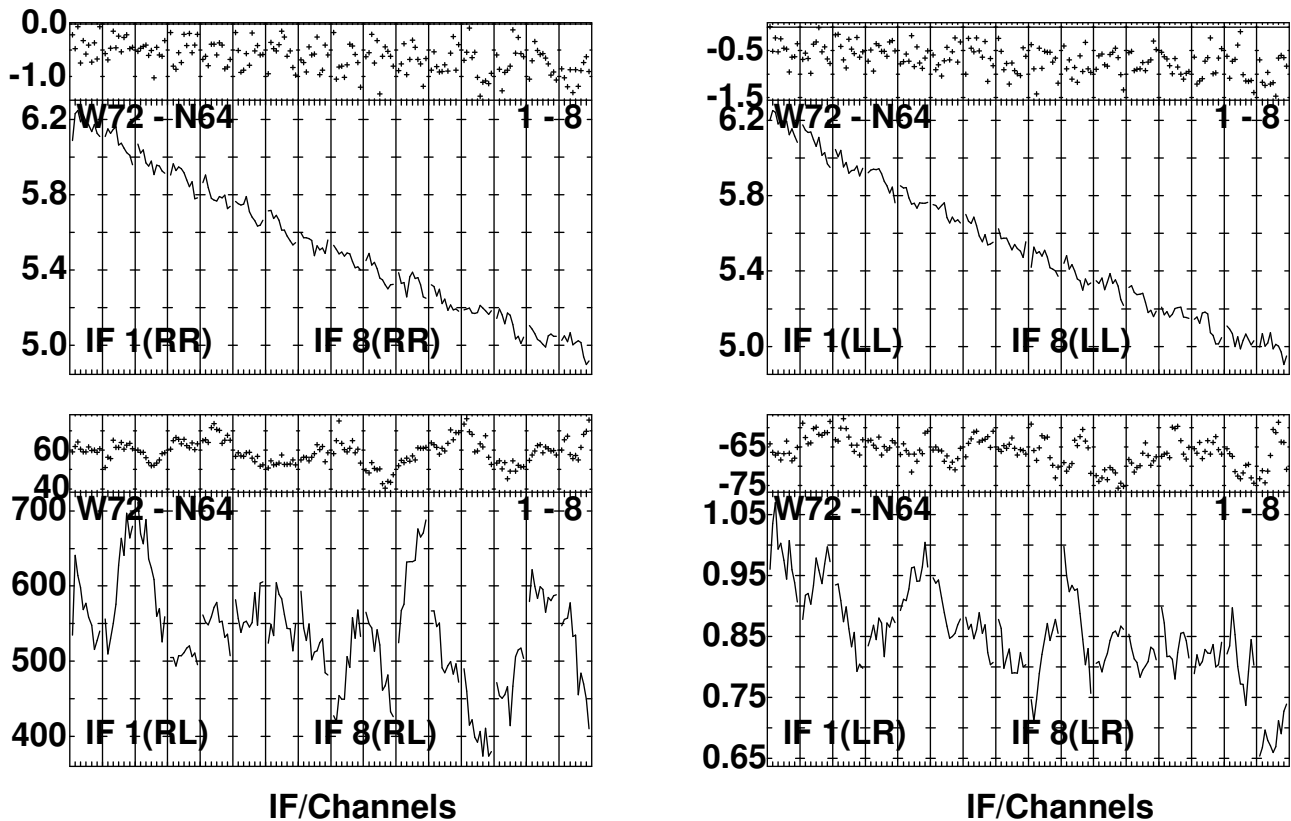A single, identical record is written for each antenna.

Fig. 2.   Data shown in Figure 1 but with the R–L delay and phase offsets applied. The strongly polarized source 3C286 (data shown) was used for the calibration. Note: the parallel–hand data is unaffected. Instrumental polarization corrections have not been applied.

VII. TESTING

The calibration data from an EVLA observation was used to test this technique. The data included the bandpass 6.0–8.0 GHz in 16 spectral windows (IFs) each averaged to 16 channels. The observations included 5×15 second scans over roughly 5 hours on the strongly linearly polarized calibrator 3C286. The EVLA has detectors sensitive to circular polarization and alt-az antenna mounts.

Data were initially calibrated using the standard parallel–hand techniques; a sample of this data on one of the longer baselines is shown in Figure 1. No corrections were applied for parallactic angle. As 3C286 passes nearly through the zenith at the EVLA, observations past transit were nearly at the same parallactic angle and were used in the fitting.

The data were obtained in the "A" configuration for which the synthesized beam is about 0.25" FWHM. 3C286 is well resolved in this configuration but the bulk of the emission, both total and polarized intensity is from the marginally resolved core. Therefore, all baselines were used for the fitting. A target R-L phase of $66°$ and rotation measure of 0 rad/m were provided. The outer 3 channels on each end of each IF were omitted from the fitting of the 3C286 data.

The cross–polarized fit was applied using CLCor and the data recalibrated. The same data shown in Figure 1 after this recalibration are shown in Figure 2. The rapid variation of the cross–polarized phase with frequency is largely eliminated. Variations of phase with frequency remaining are due to the frequency variable instrumental polarization which has not

been corrected. The fluctuations of instrumental polarization are also visible in the cross–polarized amplitudes. Note that the parallel–polarized data are unaffected by the cross–polarized calibration.

In order to test the effect of a weakly polarized calibrator, the same test was run but using as calibrator J1504+1029 whose polarization is less that the typical baseline instrumental polarization. A plot of the data shown in Figures 1 and 2 is given in Figure 3.

VIII. DISCUSSION

A relatively general and robust method for fitting cross–polarized delays and phases is presented and tested using EVLA data. In these tests, the large phase slope in the cross–polarized spectra due to the difference in the parallel–hand delays is essentially removed.

The fitting of data from circularly polarized detectors and a strongly linearly polarized calibrator, as in the first test presented, is relatively straightforward as all baselines are measuring essentially the same values and much averaging can be incorporated. More extensive testing will be needed in the case of weakly polarized calibrators where the cross-polarized visibilities will be dominated by instrumental polarization and arrays using linearly polarized detectors for which time averaging need be more limited.

A test on the same dataset as discussed above using a calibrator whose polarized emission is less than the typical baseline instrumental polarization resulted in cross–polarized
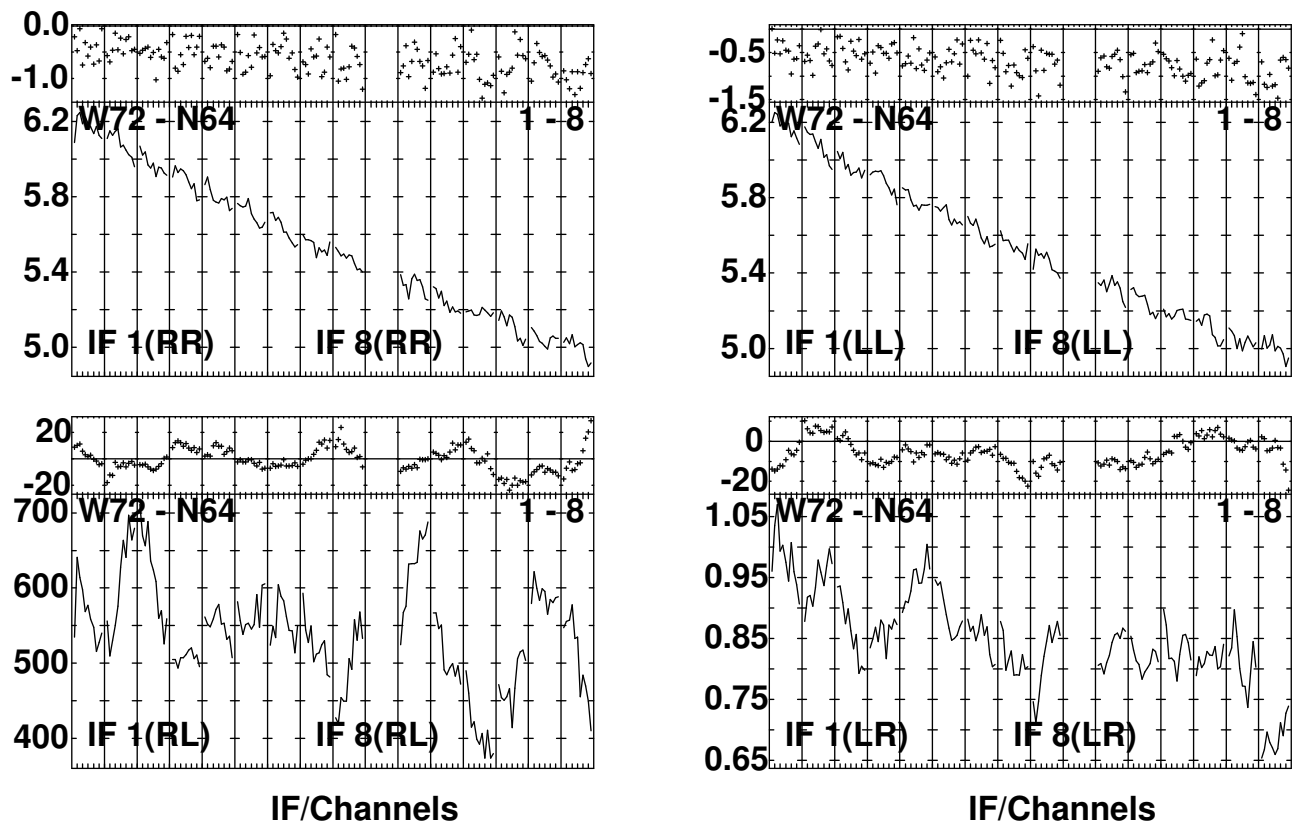
Fig. 3.   Data shown in Figure 2 but with the R–L delay determined from the weakly polarized calibrator J1504+1029.

delay fits comparable to those derived from 3C286 although with larger scatter in the R–L phase.

Fitting for cross–polarized delays is needed prior to fitting for instrumental polarization. Fitting for cross–polarized phases at this point will be only approximate due to corruption by the instrumental polarization and will need to be redetermined after removal of the instrumental polarization.

The technique presented here is available for use in the Obit package as task RLDly.

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