Tiling the Field of View with Facets

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***Abstract*—This memo discusses the layout of a mosaic of tiles covering the desired field of view when using faceting to deal with the non-coplanarity problem in radio interferometry. A scheme is described to fully cover a region with a minimum number of tiles.**

***Index Terms*—Interferometric imaging**

I. Introduction

IMAGING the celestial sphere with a interferometer array whose elements are not confined to a plane encounters problems projecting the curved sky onto flat images [1]. There are several solutions to this problem one of which is tiling the sky with facets each of which is small enough that the three dimensional nature of the sky is not an issue. An implementation of tiling in the Obit package ([2], http://www.cv.nrao.edu/∼bcotton/Obit.html) is discussed in the following.

II. The Non–coplanarity Problem

Interferometers using earth rotation synthesis measure the spatial coherence function ("visibililties") in the three dimensional space described by the vector (u,v,w). At the very large distances of most astronomical objects, the sky brightness can be considered projected onto the "celestial sphere". Deriving flat images of this curved plane is discussed in detail in [1]. One solution to this problem is to divide the sky into "facets", each of which is small enough not to be seriously affected by the curvature of the sky but, in the aggregate, tile the region of the sky of interest. In the following, this region is referred to as the "Field of View" or "FOV". The simplest variant of faceted imaging is to make each facet tangent to the celestial sphere at its center. A computationally more efficient scheme of re-projecting onto a common tangent plane is discussed in [3].

III. Faceting

One major issue with faceting is the definition of a mosaic of tiles which completely cover the FOV but with a minimum of overlap which reduces the number of facets needed, hence computational expense.

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A. Facet size

The "w" component of the interferometric baseline vector is in the pointing direction; a range of value of this component effectively gives resolution in the direction of the target field. The tangent plane of a given facet will deviate from the celestial sphere away from the tangent point resulting in defocusing of celestial objects. The usable size of a facet is then limited by the amount of defocusing acceptable.

Defocusing increases with the square of the distance from the phase tracking center and the usable area can be described as a circular region. The radius in radians of the usable part of a facet is derived in [4] as

$$\theta \ = \frac{1}{3}\sqrt{\theta_{hpbw}}$$

where θ_{hpbw} is the half power synthesized beam width. This derivation assumed that the observations included "w" values comparable to the maximum baseline length; if the maximum w is less than that, the expression becomes:

$$\theta \ = \frac{1}{3}\sqrt{\theta_{hpbw}}\ \sqrt{\frac{(Max(\sqrt{u^2+v^2})}{Max(w))}}. \qquad (1)$$

An approximation for θ_{hpbw} in radians is given by the minimum fringe spacing:

$$\theta_{hpbw} \ \approx \ \frac{1}{Max(\sqrt{u^2+v^2})}. \qquad (2)$$

B. Tiling the Field of View

The curvature of the celestial sphere necessitates defining the mosaic of facets in terms of direction cosines, l=RA direction, m=declination direction. These are defined on a tangent plane and avoid complications like cos(declination) factors. This is especially important at lower frequencies where images can cover substantial regions of the sky. Equation 1 gives the radius on the sky of the usable portion of the facet; for direction cosines, this becomes

$$R \ = \ \frac{\theta}{1 \ + \ \tan\theta}$$

The objective of defining a mosaic of tiles is to completely cover the region of interest using the minimum number of tiles. This requires minimizing the overlap among facets. This condition can be met by a hexagonal pattern of facets suitably spaced. A hexagonal pattern can be formed by centering tiles on alternate cells of a rectangular grid; this pattern should alternate between adjacent rows of the grid. The grid should be sufficiently extensive to fully cover the desired field of view.

A simple tiling is using a grid spacing in both dimensions of the radius (R) of the usable part of the facet. Such a tiling

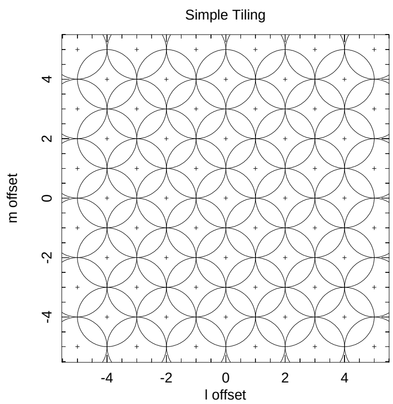

Fig. 1. Tiling using spacings in both l and m of the radius of the usable portion of a facet.

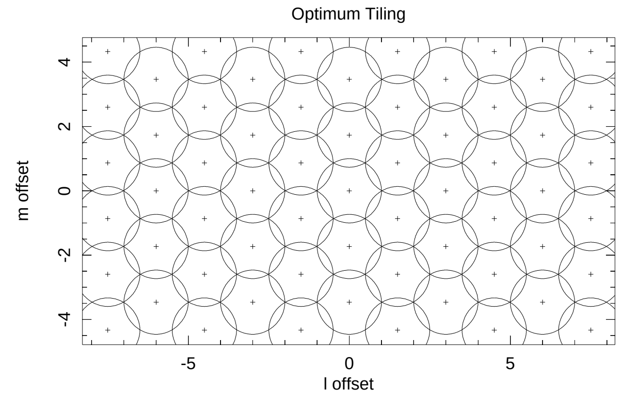

Fig. 2. Tiling using optimum spacings in l (1.5) and m (0.866).

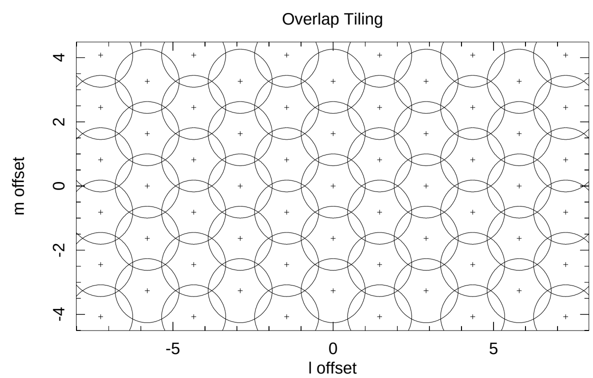

Fig. 3. Tiling including 5% overlap.

is shown in Figure 1; this tiling includes substantial overlap among tiles but mainly from only four of the six surrounding facets.

A reduced total overlap can be achieved by each of the facets surrounding a given facet having the same overlap while completely covering the FOV. This requirement means that the six surrounding facets all have the same distance from the facet in question. This puts constraints on the spacing of the grid of facets in the two celestial dimensions. If R is the radius of the usable region in a facet then the spacing in l (dx) and m (dy) can be defined as

$$dx \;=\; f_x R,\; dy \;=\; f_y R$$

In a hexagonal grid, there are surrounding facets with two types of orientations, 1) north and south of the facet in question or 2) at ±45° on either side The center-to-center distance of the north/south facets is $2\ f_y\ R$ and the distance to the 45° facets is $\sqrt{(f_x\ R)^2 + (f_y\ R)^2}$. Setting these two distances equal gives:

$$2\ f_y\ R \;=\; \sqrt{(f_x R)^2 + (f_y\ R)^2}$$

or

$$3\ f_y^2 \;=\; f_x^2 \tag{3}$$

Another possible condition is that each of the usable regions of the outlying facets intersect the usable region of the central facet at locations 30° from the cardinal points. This will guarantee complete coverage. One of these points is $(R\ \sin(30),\ R\ \cos(30))$. A nearby 45° facet is located at an offset $(R\ f_x,\ R\ f_y)$. This adds the constraint that

$$(f_x - \sin(30))^2 + (f_y - \cos(30))^2 \;=\; 1 \tag{4}$$

The solution of these two conditions (Eqn, 3 and 4) gives

$$f_x \;=\; 1.500, f_y \;=\; 0.866$$

or

$$f_x \;=\; 1 + \sin(30), f_y \;=\; \cos(30) \tag{5}$$

A tiling using these spacings is shown in Figure 2; this tiling has less overlap while completely covering a larger area.

In practice, complete coverage is inadequate as it is desirable that each region of emission appear away from the edge of the usable region in at least one facet. It is thus desirable to incorporate a minimum extra overlap between facets. A tiling like Figure 2 but including a 5% extra overlap is shown in Figure 3.

C. Conversion to Equatorial Coordinates

The tiling described above is defined in terms of direction cosines (l,m) about the pointing position (α_0,δ_0). The conversion to celestial coordinates (α,δ) for a given facet using the Sine projection is given by equations 6 and 7.

$$\alpha \;=\; \alpha_0 + \mathrm{atan2}(l,\ \cos\delta_0\sqrt{1 - l^2 - m^2} + m\ \sin\delta_0) \tag{6}$$

$$\delta \;=\; \sin^{-1}(\sin\delta_0\sqrt{1 - l^2 - m^2} + m\ \cos\delta_0) \tag{7}$$

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