

GBT Performance

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GBT Performance
20 June 2006 Green Bank

Telescope Structure



Focus



Pointing



Surface



Pointing Requirements

$$g(\rho) = \exp\left[-4 \ln 2 \left(\frac{\rho}{\theta}\right)^2\right]$$

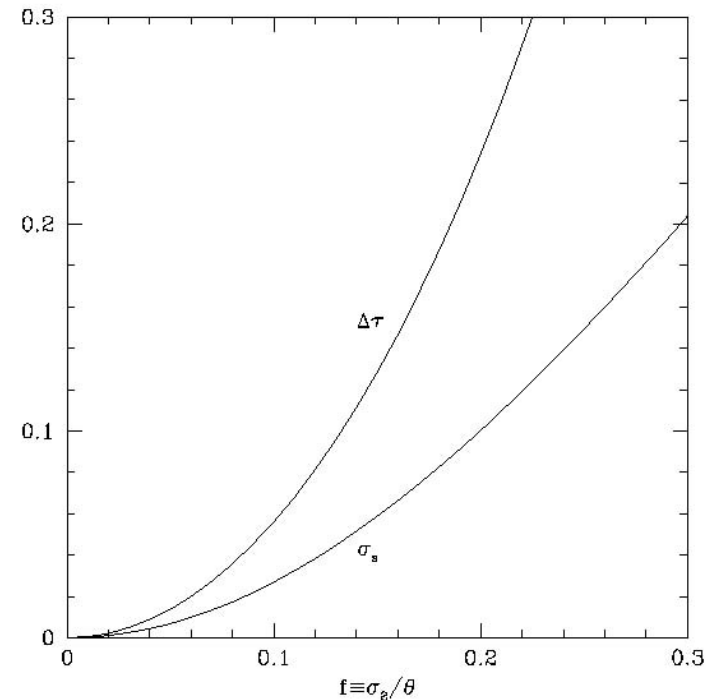
$$\left(\frac{\theta}{740 \text{ arcsec}}\right) \approx \left(\frac{\text{GHZ}}{\nu}\right)$$

$$\langle \rho^2 \rangle \equiv \sigma_2^2 = \sigma_{\text{Az}}^2 + \sigma_{\text{El}}^2$$

$$f \equiv \left(\frac{\sigma_2}{\theta}\right)$$

Good ($\sigma_s = 5\%$) $\rightarrow f \approx 0.14$

Usable ($\sigma_s = 10\%$) $\rightarrow f \approx 0.20$



Focus Requirements

$$g_a = \exp \left[-4 \ln 2 \left(\frac{\Delta y_s}{\theta_a} \right)^2 \right] \quad \text{Axial}$$

$$\theta_a \approx 4\lambda$$

$$\text{Good } (g_a > 0.99) \rightarrow \Delta y_s < \theta_a / 16 < \lambda / 4$$

$$\text{Usable } (g_a > 0.95) \rightarrow \Delta y_s < \theta_a / 8 < \lambda / 2$$

$$g_l = \exp \left[-4 \ln 2 \left(\frac{\Delta x_s}{\theta_l} \right)^2 \right] \quad \text{Lateral}$$

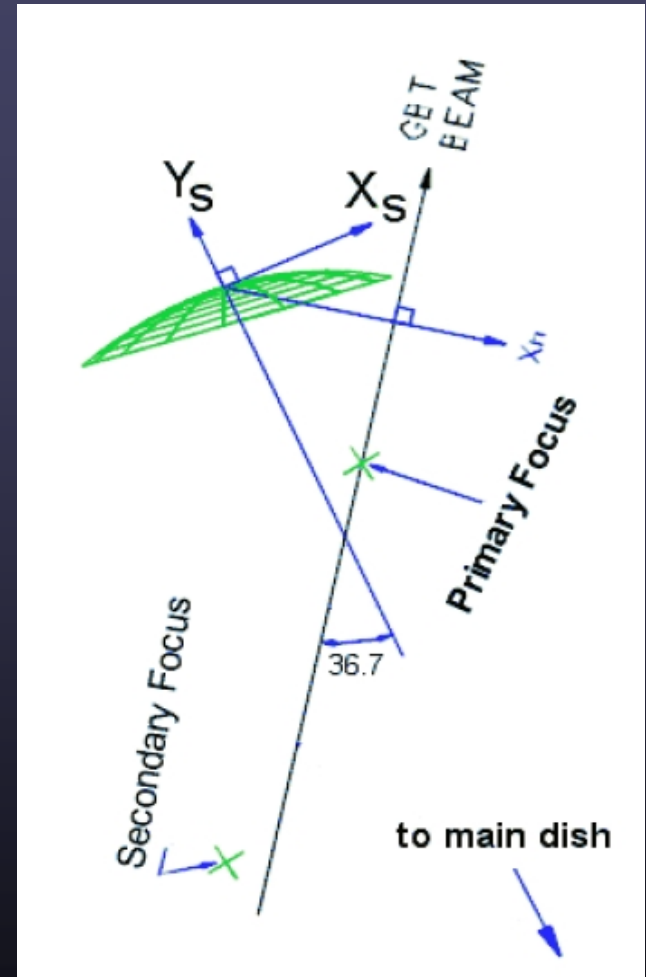
$$\theta_l \approx 6\lambda$$

$$\text{Good } (g_l > 0.99) \rightarrow \Delta x_s < \theta_l / 16 < \lambda / 3$$

Plate Scale $\approx 3.7'' / \text{mm}$

$$\text{Q-band: } \lambda = 7 \text{ mm} \rightarrow \Delta x_s < 2.3 \text{ mm}$$

$$\rightarrow \theta = 17.3'' \rightarrow f = 0.5$$





Surface Requirements

$$\eta_s \approx \exp\left[-\left(\frac{4\pi\varepsilon}{\lambda}\right)^2\right]$$

η_s is the aperture efficiency for the surface.

ε is the rms surface error.

$$S_\nu = \frac{2kT'_A}{\eta_a A_g}$$

S_ν is the flux density.

k is Boltzmann's constant.

T'_A is the antenna temperature corrected for the atmosphere.

η_a is the aperture efficiency.

A_g is the geometric primary area.

Good : $\varepsilon < \lambda / 16 \rightarrow \eta_s \approx 0.54$

Usable : $\varepsilon < \lambda / 4\pi \rightarrow \eta_s \approx 0.37$



Repeatable Errors – Pointing

- Telescope misalignments:
 - Azimuth track tilt
 - Horizontal collimation
 - Elevation axle collimation
 - Encoder offsets
- Gravity: assume linear elastic structure

$$\Delta A \cos(E) = d_{0,0} + b_{0,1} \sin(E) + d_{0,1} \cos(E) + \\ b_{1,1} \cos(A) \sin(E) + a_{1,1} \sin(A) \sin(E)$$

$$\Delta E = d_{0,0} + c_{1,0} \sin(A) + d_{1,0} \cos(A) + b_{0,1} \sin(E) + d_{0,1} \cos(E)$$

Repeatable Errors – Focus

- Gravity: assume linear elastic structure

$$X = A_x + B_x \text{Cos}(E) + C_x \text{Sin}(E)$$

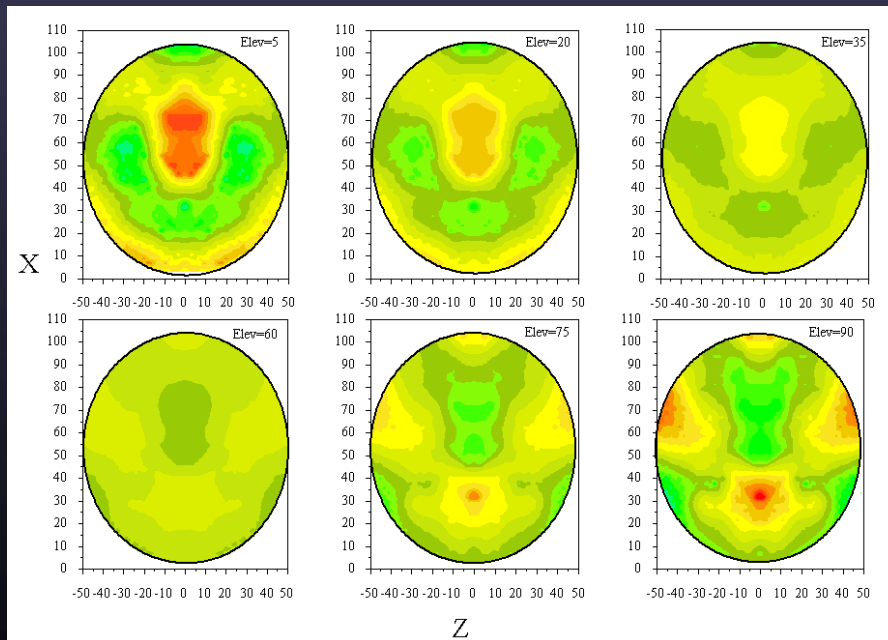
$$Y = A_y + B_y \text{Cos}(E) + C_y \text{Sin}(E)$$

$$Z = A_z + B_z \text{Cos}(E) + C_z \text{Sin}(E)$$

Repeatable Errors – Surface

- Gravity:
 - Homologous design (almost)
 - Photogrammetry (50.3 deg) → Zero-point
 - Finite Element Model (FEM)
 - Best Fit Parabola (BFP)

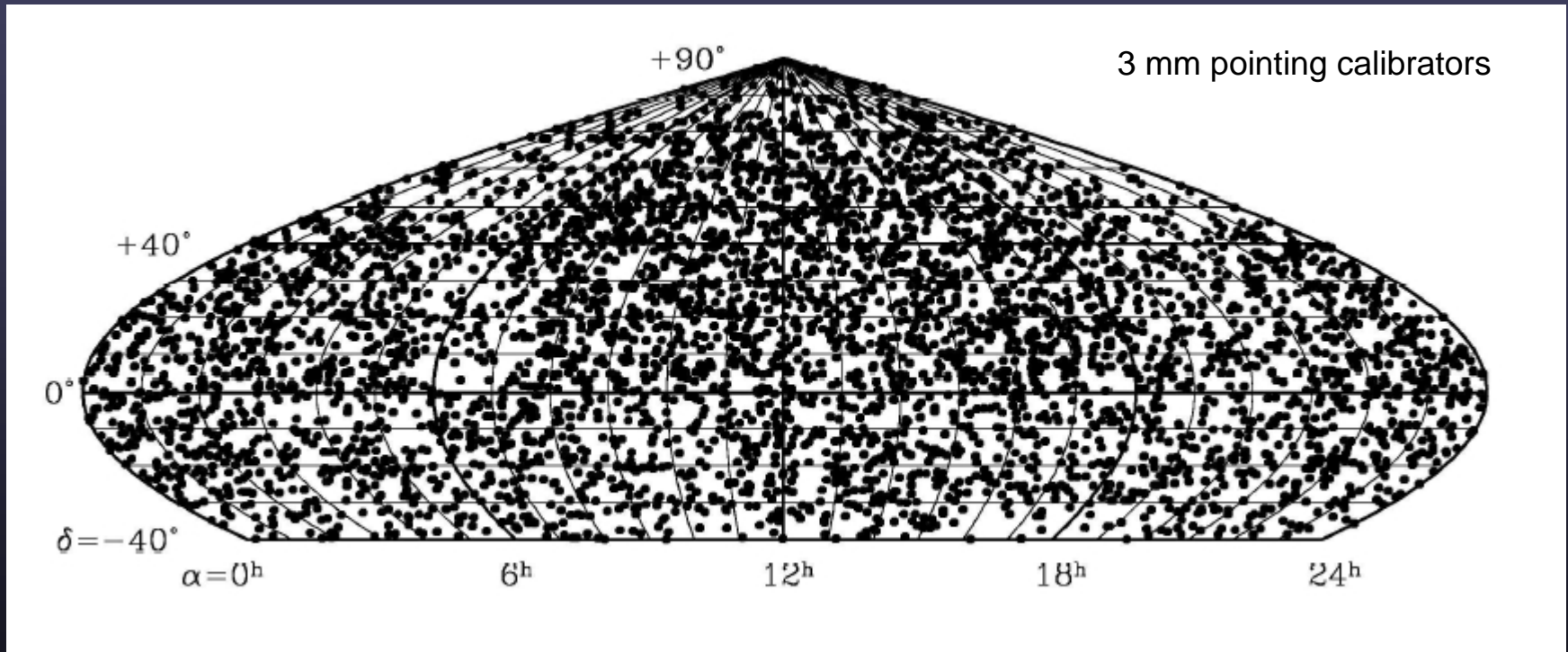
Active Surface: 2004 Panels
2209 Actuators



Point Source Calibrators



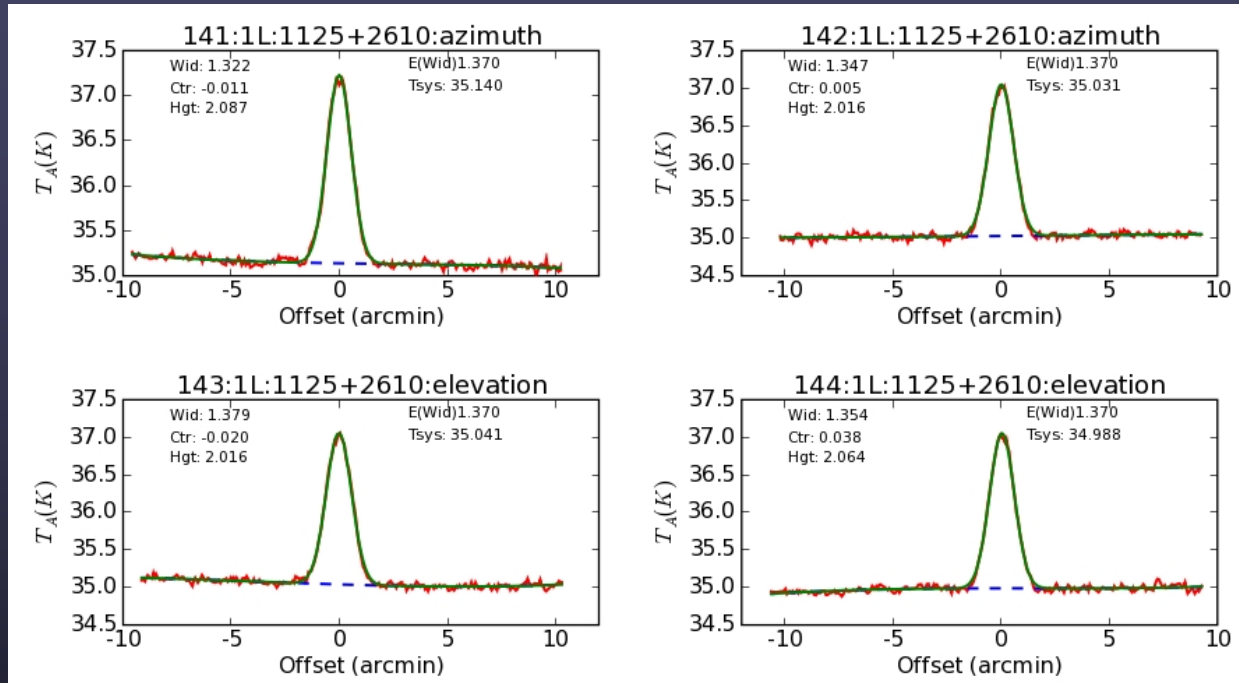
PCALS 4.0: 7108 sources
Two-dimensional rms error < 0.2 arcsec



3 mm pointing calibrators

Condon & Yin (2005)

Pointing Model



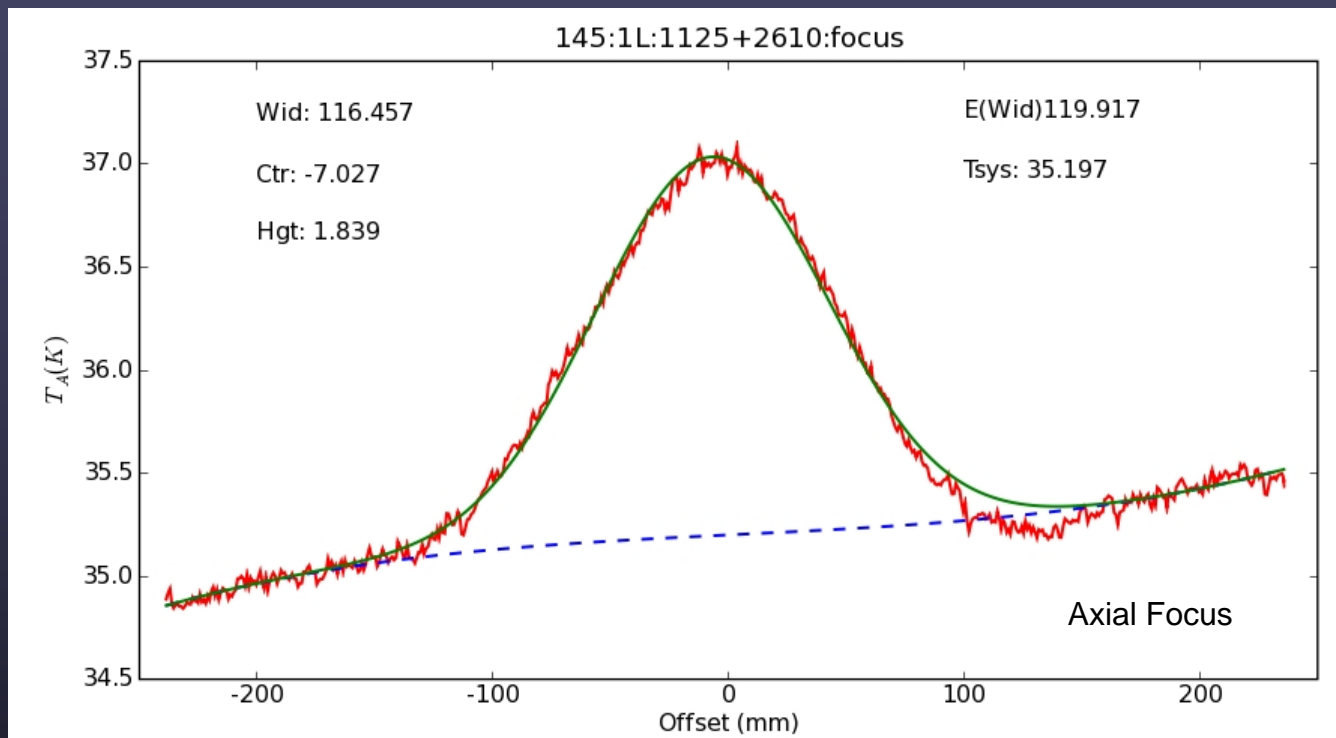
Azimuth Series $\Delta A \cos(E)$

Elevation Series ΔE

Coeff. (M&C)	Coeff. (TPOINT)	Term	Value (arcsec)	σ (arcsec)	Meaning
$d_{0,0}$	-CA	1	-29.82	6.221	Horizontal Collimation
$b_{0,1}$	-NPAE	Sin E	-3.57	4.451	El Axle Collimation
$d_{0,1}$	-IA	Cos E	-11.71	4.943	Az Zero
$b_{1,1}$	AW	Cos A Sin E	+2.20	0.422	Zenith E-Tilt
$a_{1,1}$	AN	Sin A Sin E	+2.97	0.431	Zenith N-Tilt

Coeff. (M&C)	Coeff. (TPOINT)	Term	Value (arcsec)	σ (arcsec)	Meaning
$d_{0,0}$	IE	1	-758.47	6.226	El Zero
$c_{1,0}$	-AW	Sin A	-2.20	0.422	Zenith E-Tilt
$d_{1,0}$	AN	Cos A	+2.97	0.431	Zenith N-Tilt
$b_{0,1}$	ECES	Sin E	+678.13	4.443	Asymmetric Gravity
$d_{0,1}$	ECEC	Cos E	+795.03	4.925	Symmetric Gravity

Focus Model

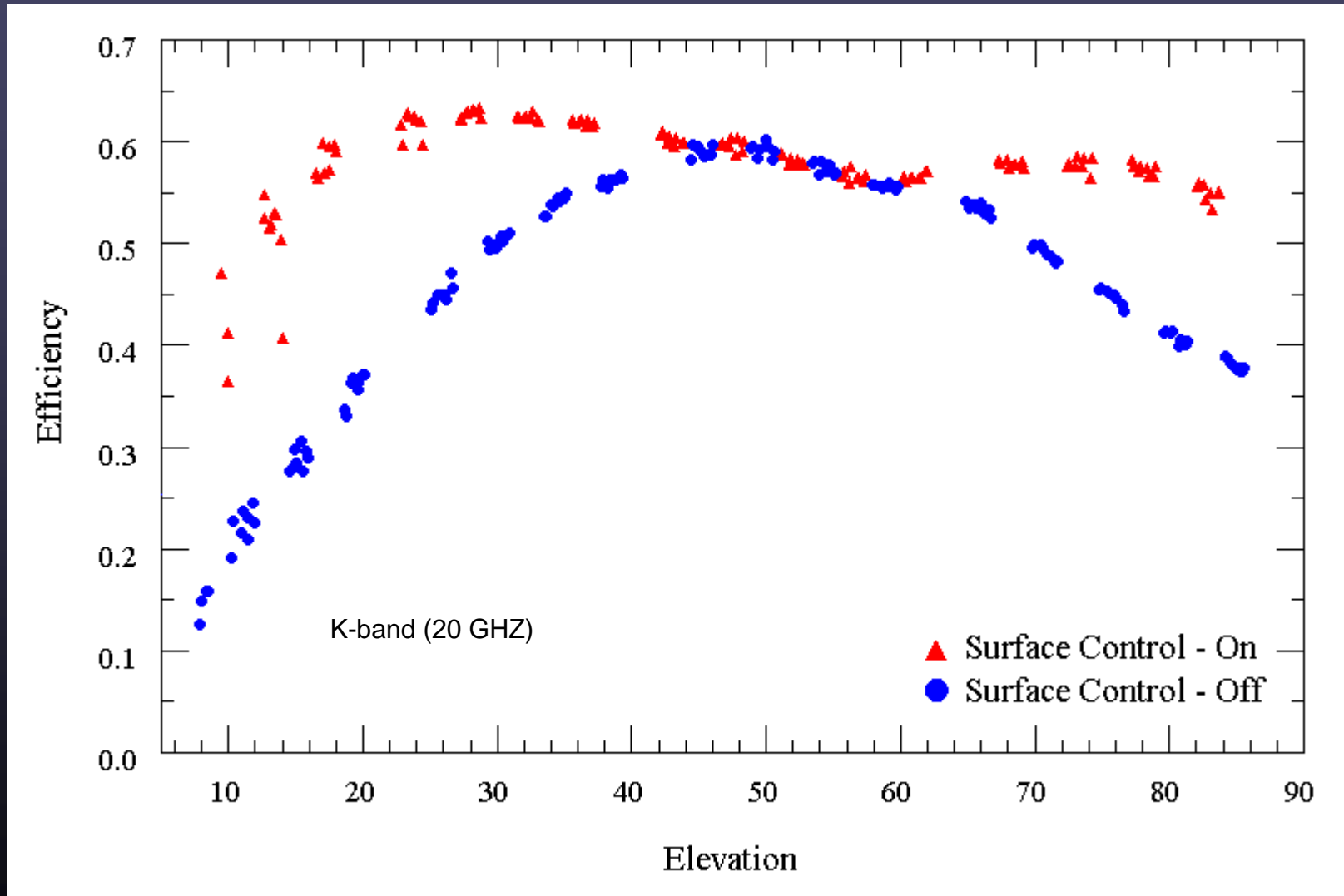


$$X = 212.55 - 301.98 \cos(E) - 25.55 \sin(E) \text{ mm}$$

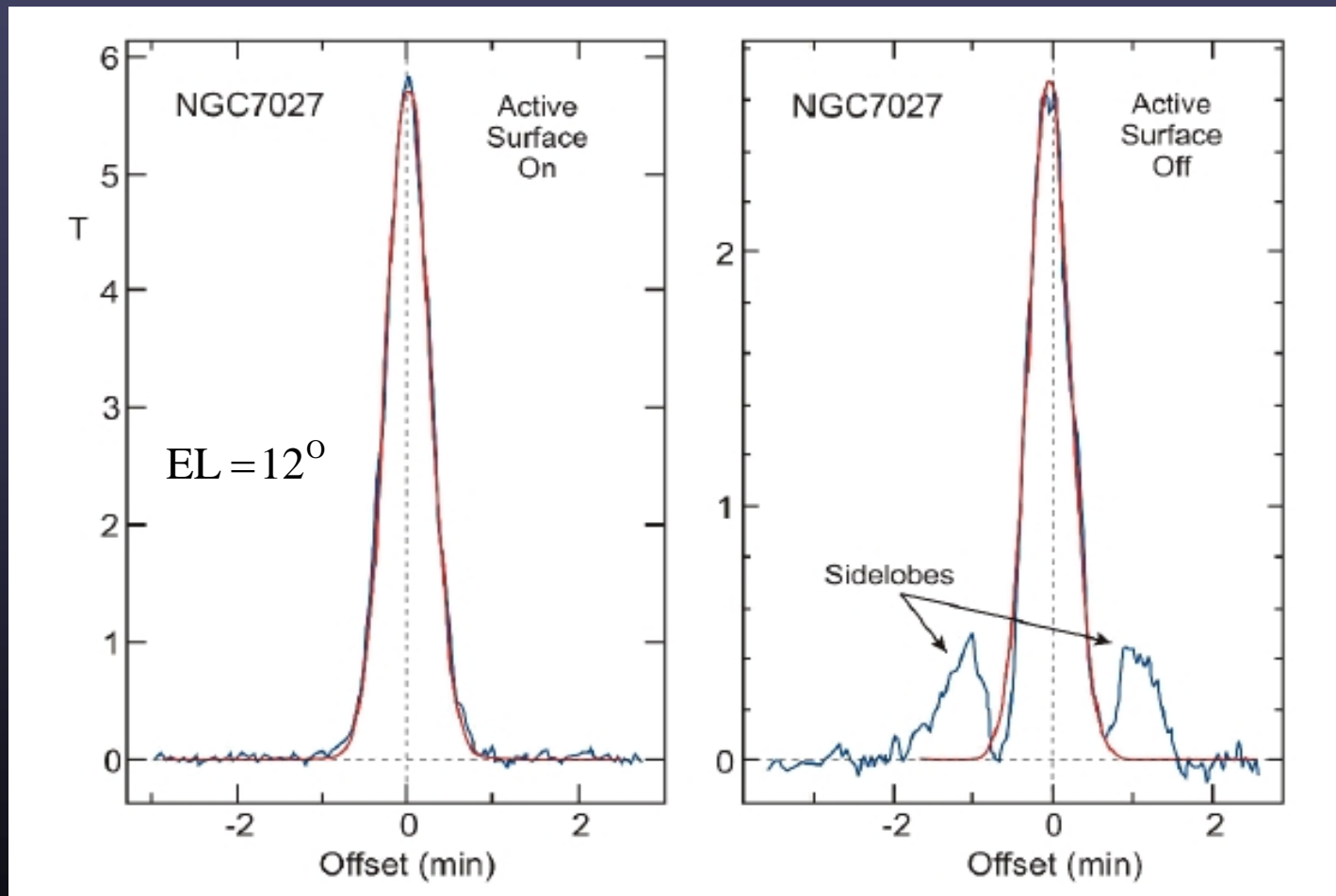
$$Y = -148.39 + 183.74 \cos(E) + 9.96 \sin(E) \text{ mm}$$

$$Z = 9.56 + 11.18 \cos(E) - 21.86 \sin(E) \text{ mm}$$

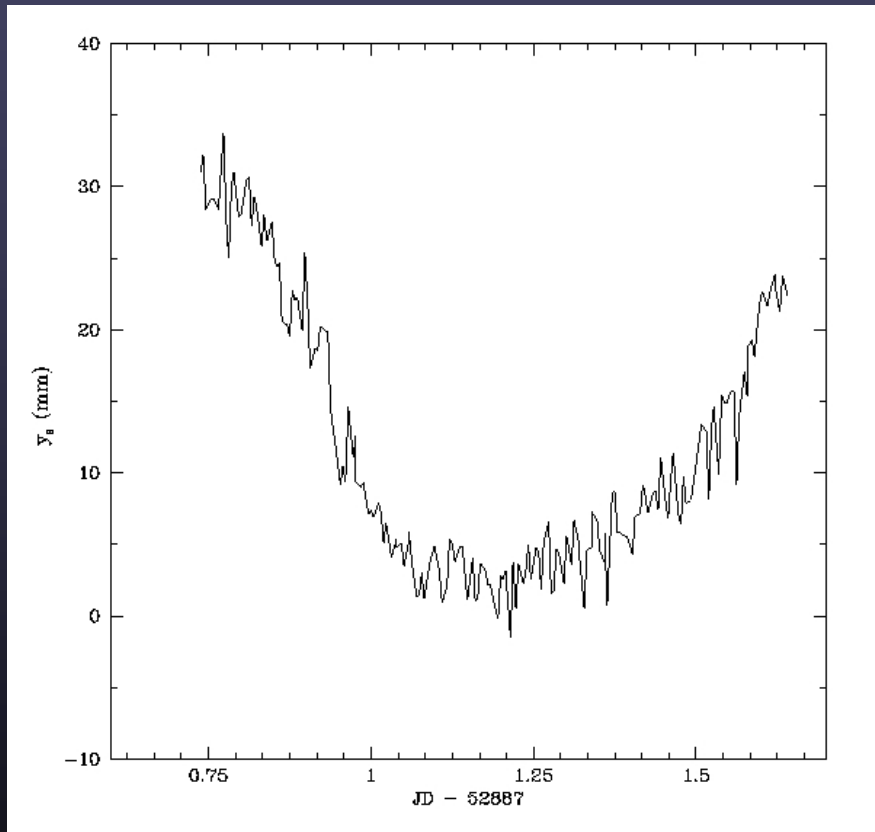
Finite Element Model – Gain Elevation Curve



Finite Element Model – Sidelobes



Non-repeatable Errors – Thermal

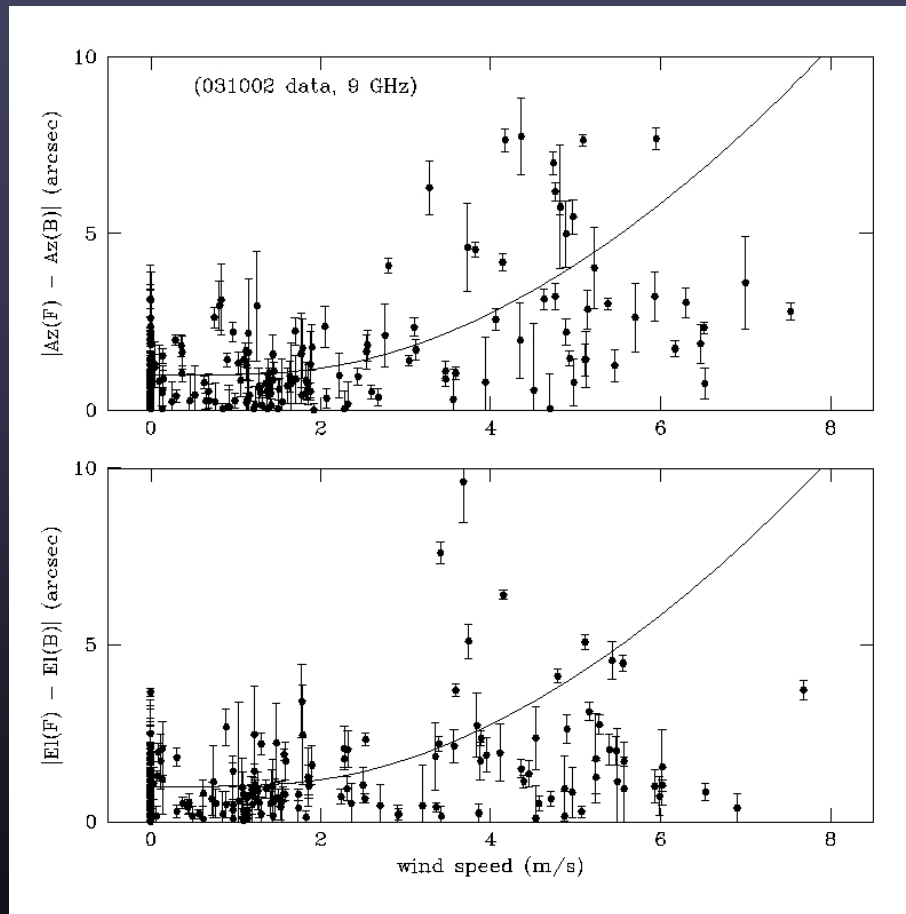


30 mm $\rightarrow \lambda/3$ at a frequency of 3 GHz

30 mm $\rightarrow 3\lambda$ at a frequency of 30 GHz

Usable ($g_a > 0.95$) $\rightarrow \Delta y_s < \theta_a / 8 < \lambda / 2$

Non-repeatable Errors – Wind

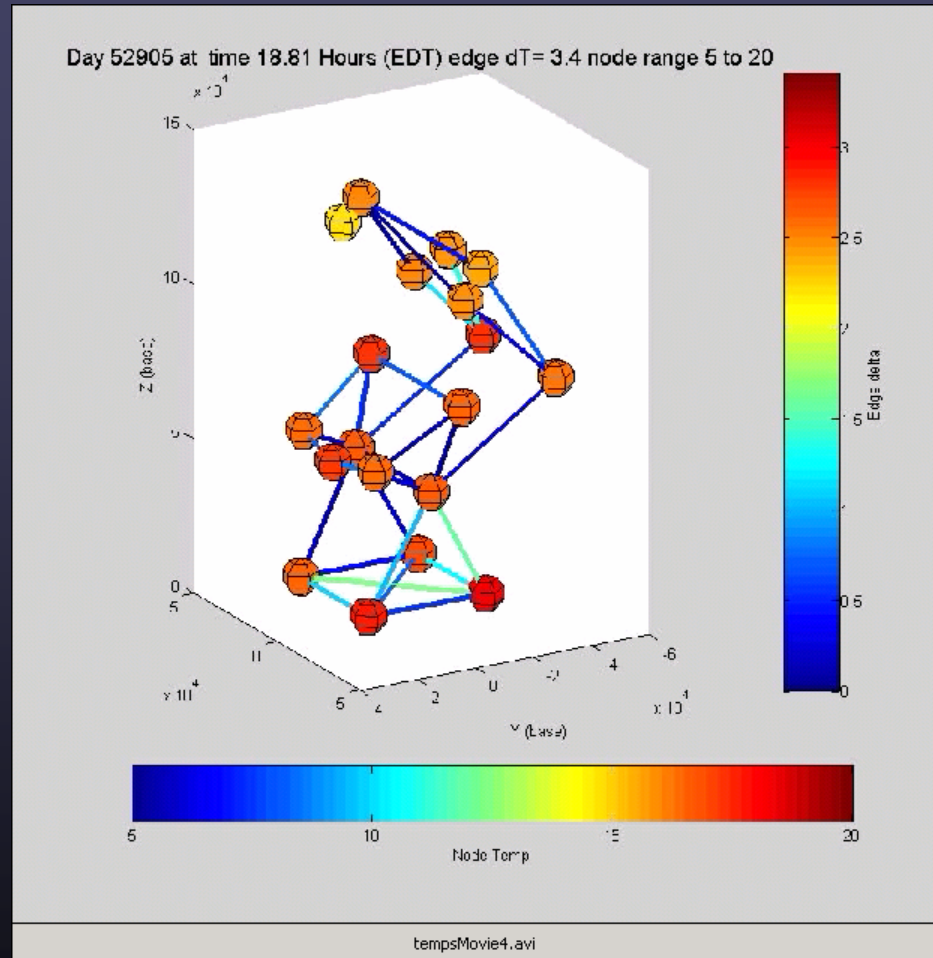


$$\sigma_1(wind) \approx 0.16 \left[\frac{s}{m s^{-1}} \right]^2 \text{ arc sec}$$

$$\sigma_2(wind) \approx \sqrt{2} \sigma_1(wind)$$

$$\approx 8'' \text{ at } s = 6 m s^{-1}$$

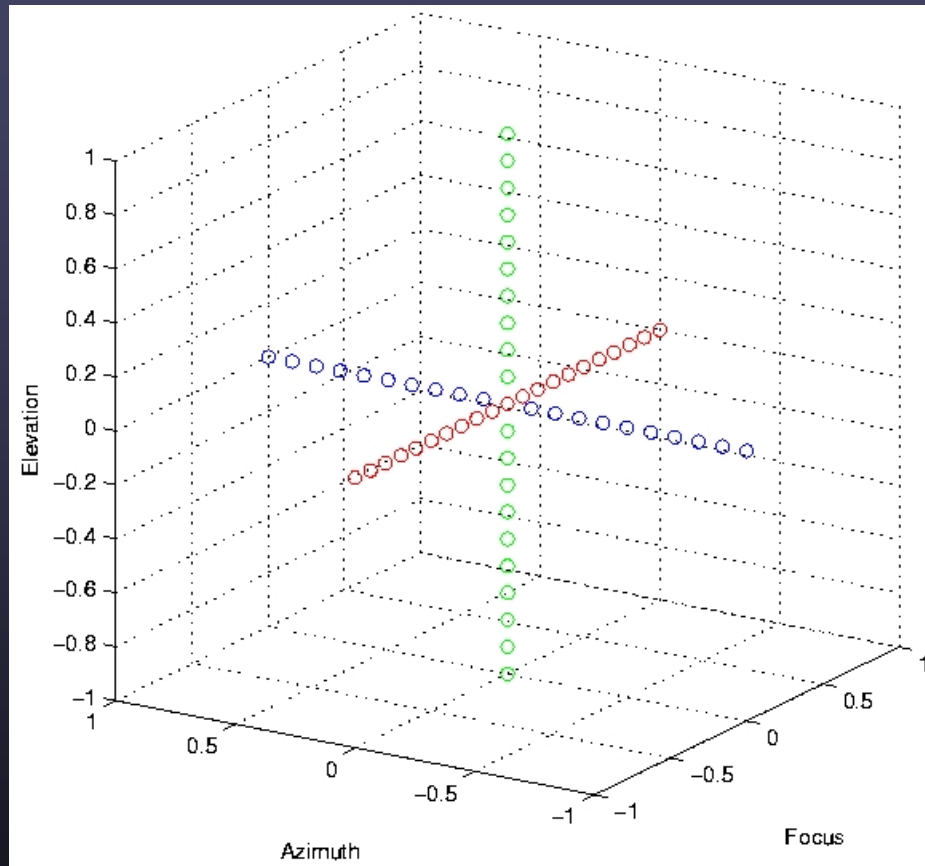
Thermal Pointing and Focus Corrections



Constantikes (2003)



Data Collection and Quality



Jack Scan

All-sky Observations
Single Source Track

Up-Down at Night → Gravity
NCP Source → Temperature

Gaussian Fits (Az, El, Focus)

Polarization (LCP – RCP)

Direction (Forward – Backward)

Thermal Model – Focus



Constantikes (2003)

$$\Delta \tilde{F}(\phi, T_i^{(f)}) = M^{(f)} \times \begin{bmatrix} T_1^{(f)} \\ \cdot \\ \cdot \\ \cdot \\ T_6^{(f)} \\ 1 \\ \sin(\phi) \\ \cos(\phi) \end{bmatrix} = M^{(f)} \times T^{(f)}$$

<u>Term</u>	<u>Coefficient</u>	<u>Min-Max</u>	<u>Significance</u>	<u>Parameter</u>
M ₁	1.086	13.1	14.3	SR-Pri
M ₂	-0.697	6.2	-4.3	VFA-Pri
M ₃	3.981	15.6	62.0	HFA
M ₄	-7.326	0.9	-6.8	BUS V1
M ₅	-0.688	12.1	-8.3	BUS V2
M ₆	-2.576	12.1	-31.2	BUS F
M ₇	-180.630	0.0	0.0	Offset
M ₈	66.189	.7	43.1	sin term
M ₉	196.949	0.6	110.8	cos term



Thermal Model – Azimuth

$$\Delta A(\phi, \theta, T_i^{(a)}) = M^{(e)} \times \begin{bmatrix} T_1^{(a)} \\ \cdot \\ \cdot \\ \cdot \\ T_4^{(a)} \\ 1 \\ \sin(\phi) \\ \cos(\phi) \\ \cos(\theta)\sin(\phi) \\ \sin(\theta)\sin(\phi) \end{bmatrix} = M^{(a)} \times T^{(a)}.$$

Constantikes (2003)

<u>Term</u>	<u>Coefficient</u>	<u>Min-Max</u>	<u>Significance</u>	<u>Parameter</u>
M ₁	5.5862	4.0	22.4	Alidade
M ₂	-8.0331	2.7	21.3	HFA
M ₃	-1.6289	2.4	3.8	BUS
M ₄	1.3683	2.0	2.8	VFA
M ₅	3.4124	0.0	0.0	CA, d(0,0)
M ₆	1.3223	0.7	1.0	NPAE, b(0,1)
M ₇	3.5152	0.9	3.0	IA, d(0,1)
M ₈	-2.4960	1.9	4.8	AW, b(1,1)
M ₀	-1.3360	1.8	2.5	AN, a(1,1)



Thermal Model – Elevation

$$\Delta E(\phi, \theta, T_i^{(e)}) = M^{(e)} \times \begin{bmatrix} T_1^{(e)} \\ \cdot \\ \cdot \\ T_4^{(e)} \\ 1 \\ \sin(\phi) \\ \cos(\phi) \\ \sin(\theta) \\ \cos(\theta) \end{bmatrix} = M^{(e)} \times T^{(e)}.$$

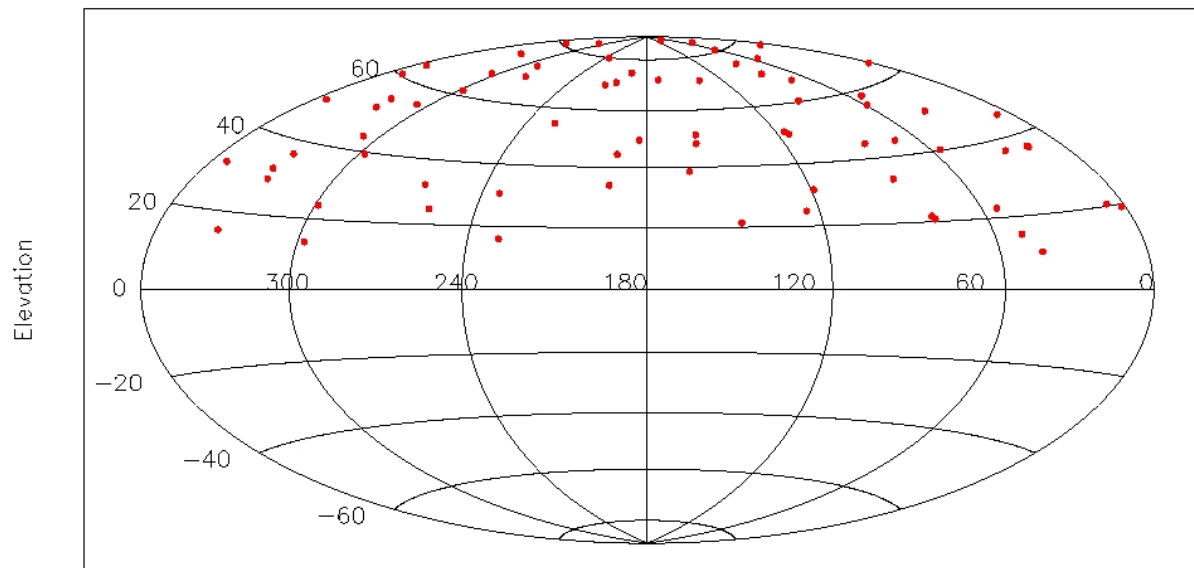
Constantikes (2003)

<u>Term</u>	<u>Coefficient</u>	<u>Min-Max</u>	<u>Significance</u>	<u>Parameter</u>
M ₁	-4.6455	1.2	-5.3	BUS
M ₂	1.7830	15.6	-27.8	HFA
M ₃	4.4488	5.9	26.4	VFA
M ₄	-8.4477	1.6	-14.0	Alidade
M ₅	62.2218	0.0	+0.000	-IE,d(0,0)
M ₆	-55.8624	0.7	-62.792	HZCZ,b(0,1)
M ₇	-22.8268	0.9	-38.216	HZSZ,d(0,1)
M ₈	2.4960	2.0	+2.169	-AW,c(1,0)
M ₀	-1.3360	2.0	-1.750	AN,d(1,0)

Performance – Blind Pointing

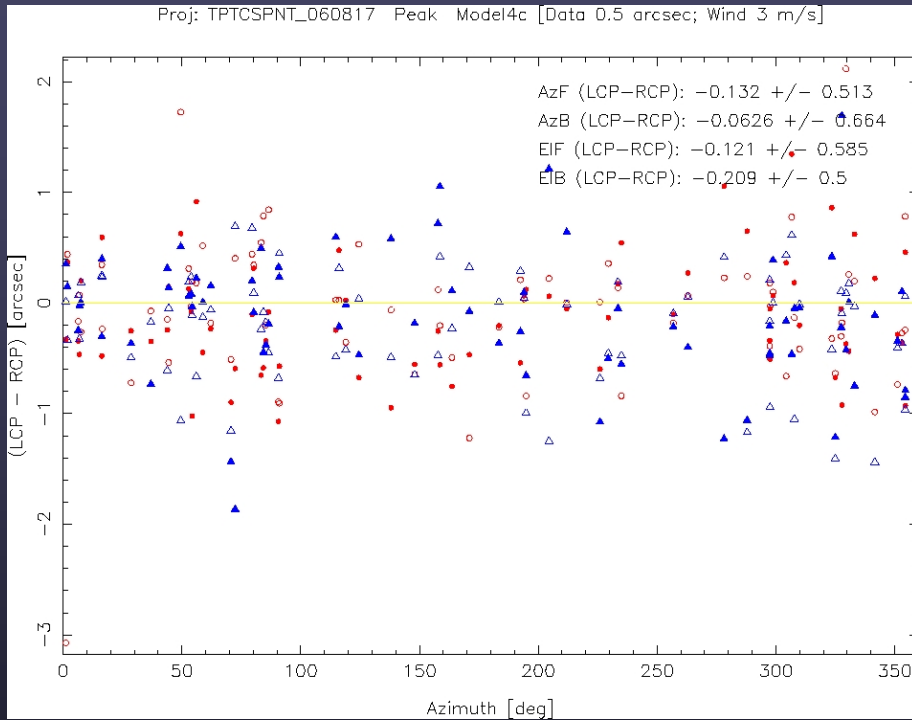


Proj: TPTCSPNT_060817 Peak Model4c [Data 0.5 arcsec; Wind 3 m/s]

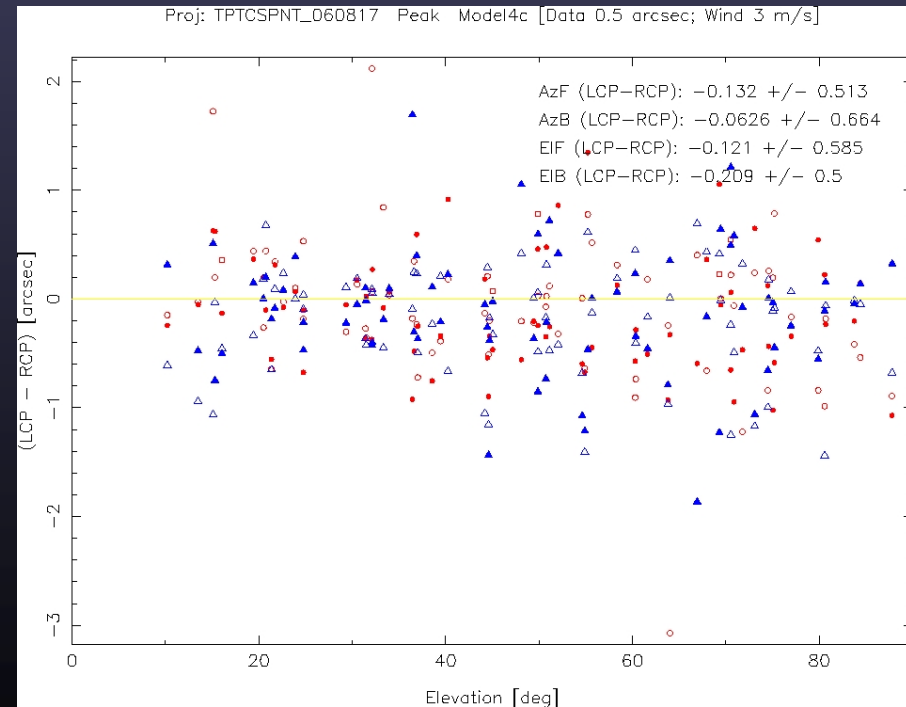


Azimuth centered on 180

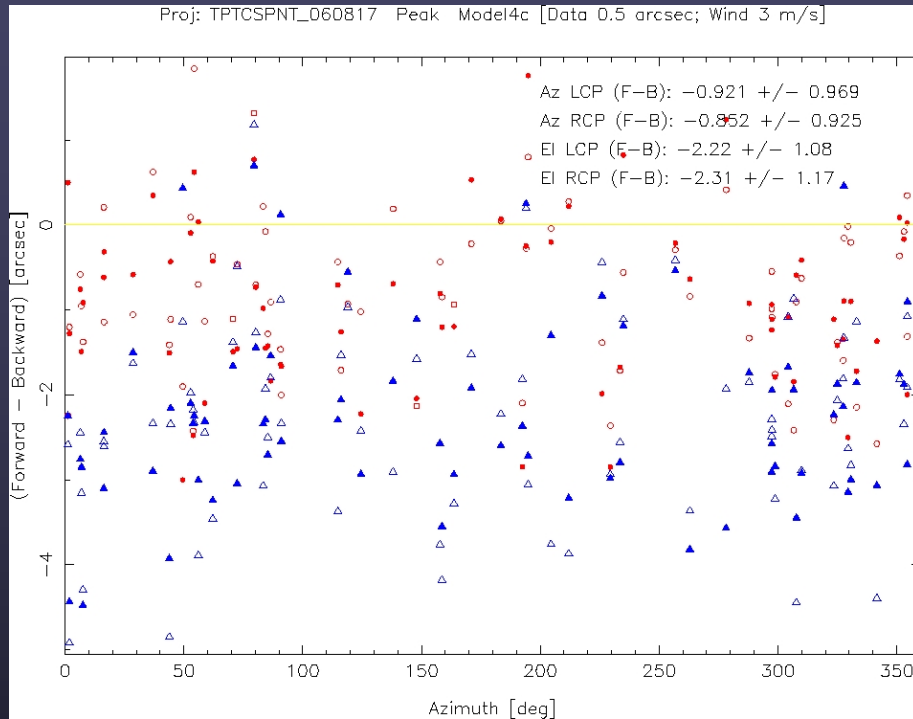
Pointing Data Quality – Polarization



$$[\text{LCP} - \text{RCP}] = -0.13 \pm 0.57 \text{ arcsec}$$

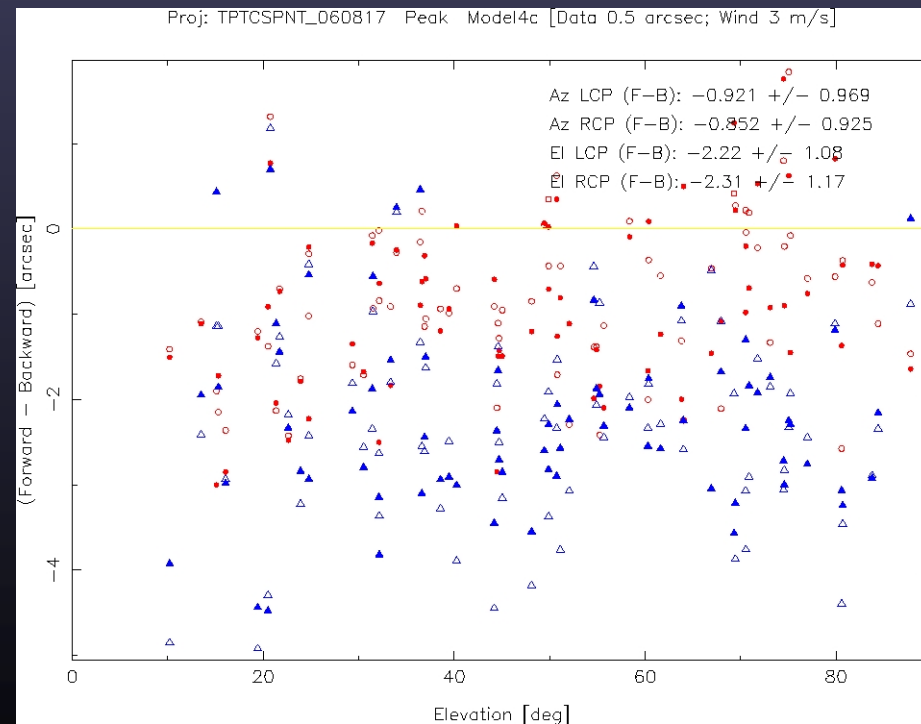


Pointing Data Quality – Direction



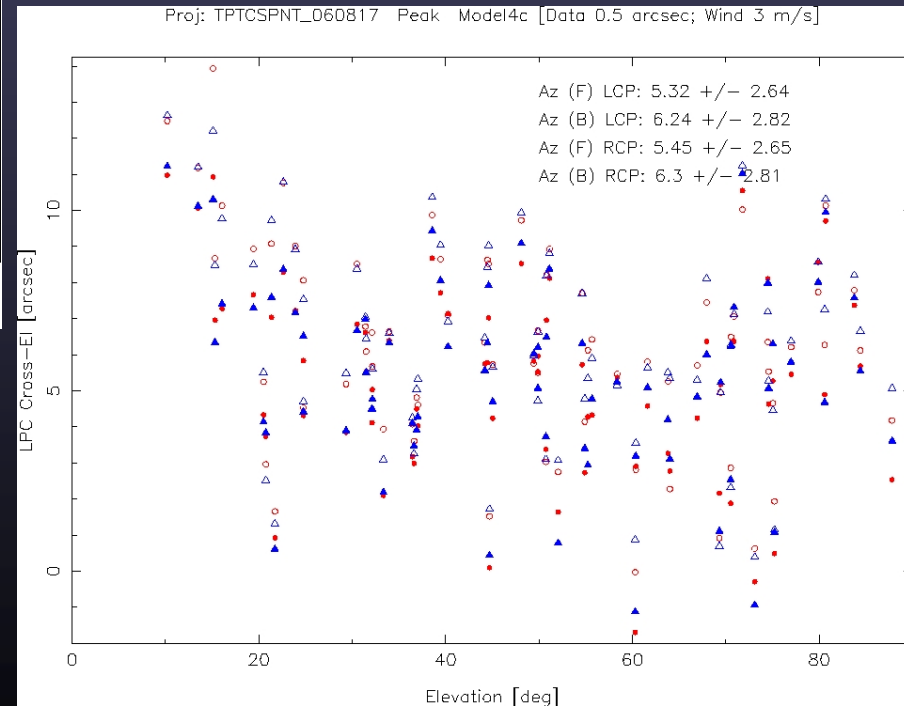
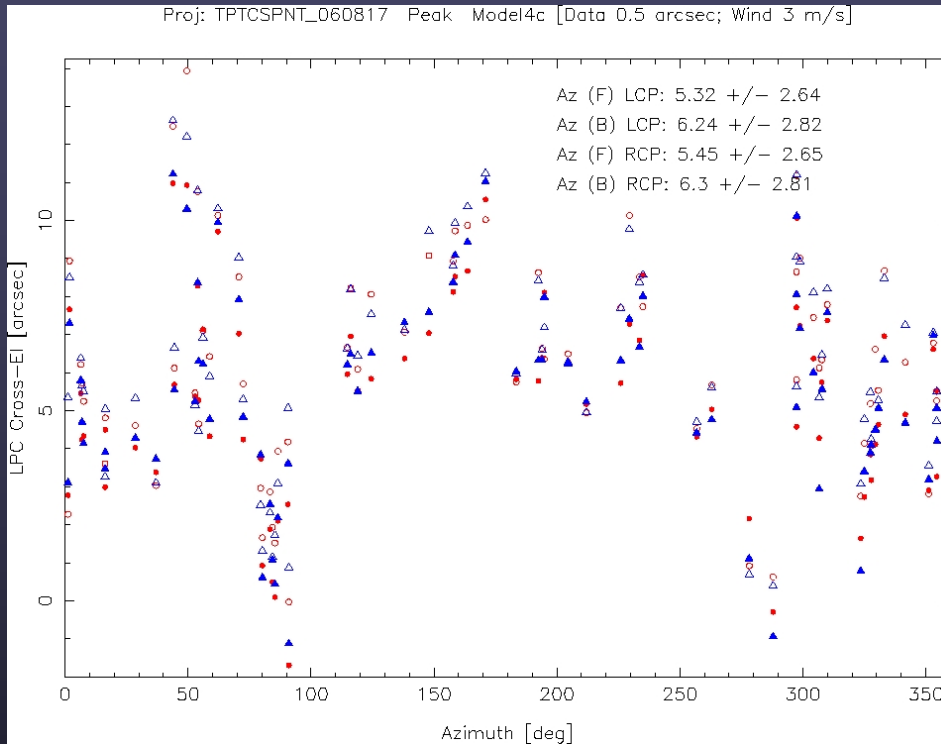
$$[F - B]_{Az} = -0.89 \text{ arcsec}$$

$$[F - B]_{El} = -2.3 \text{ arcsec}$$



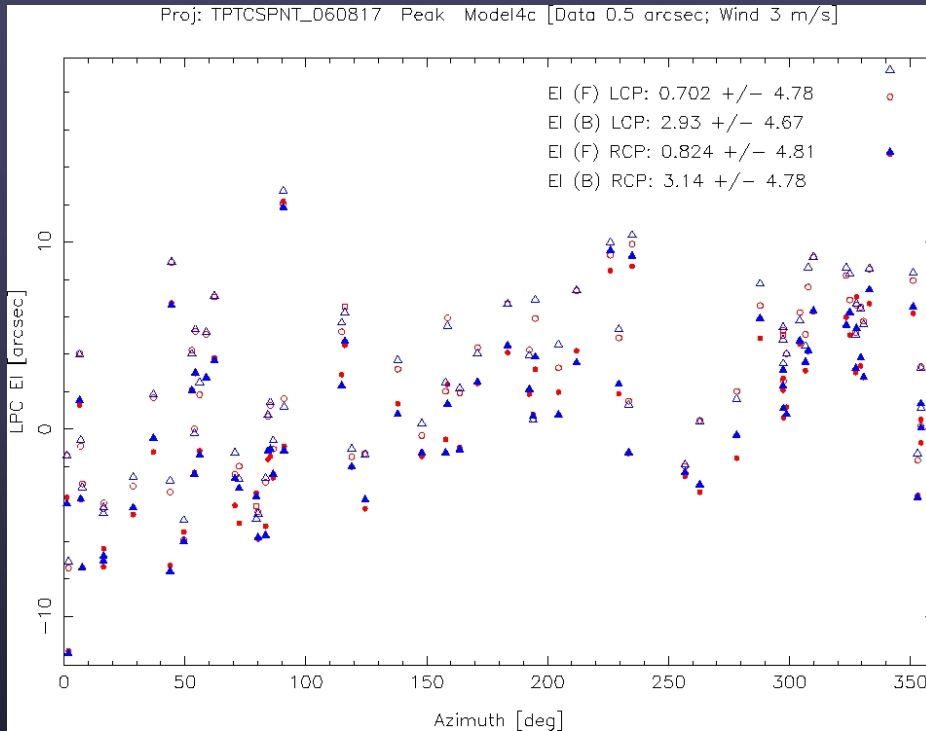


Cross-Elevation Uncertainty

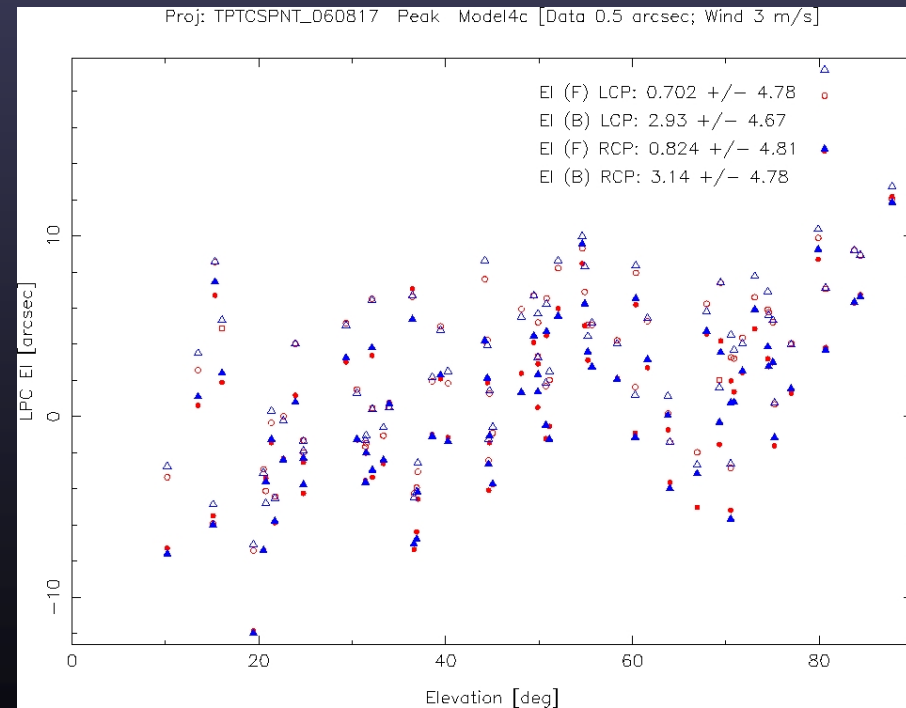


$$\sigma[\Delta A \text{ Cos}(E)] = 2.7 \text{ arcsec}$$

Elevation Uncertainty

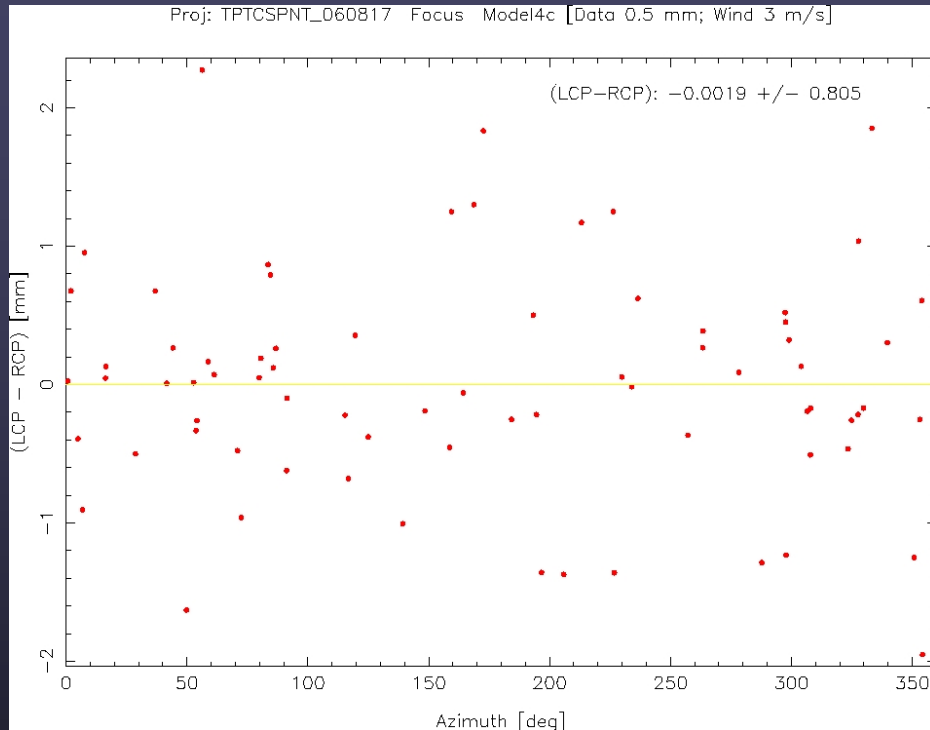


$$\sigma[\Delta E] = 4.8 \text{ arcsec}$$

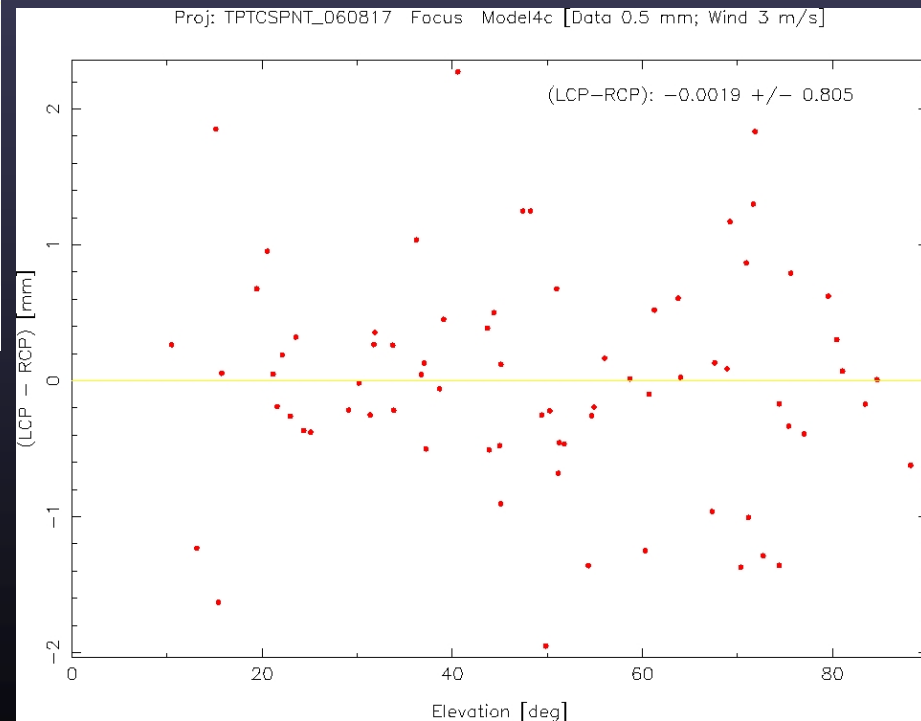




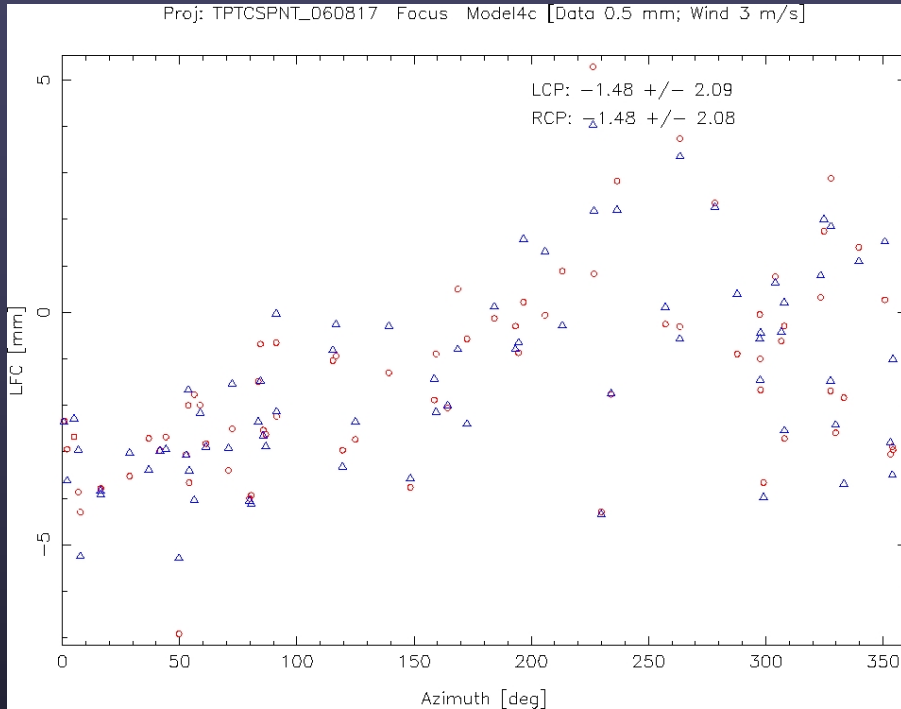
Focus Data Quality – Polarization



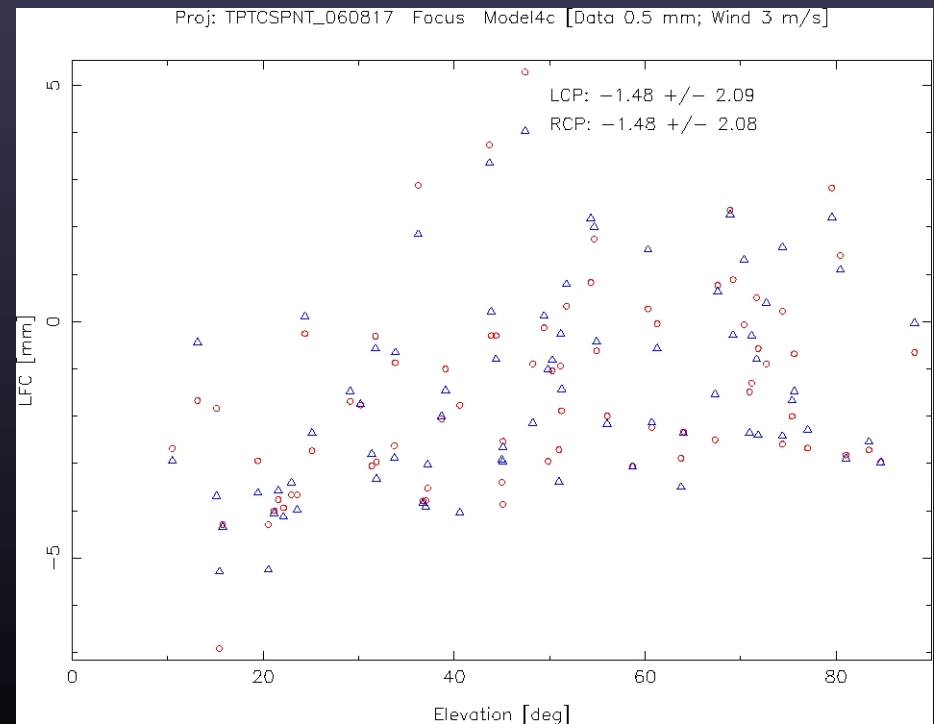
$$[\text{LCP} - \text{RCP}] = -0.0019 \pm 0.81 \text{ mm}$$



Focus Uncertainty



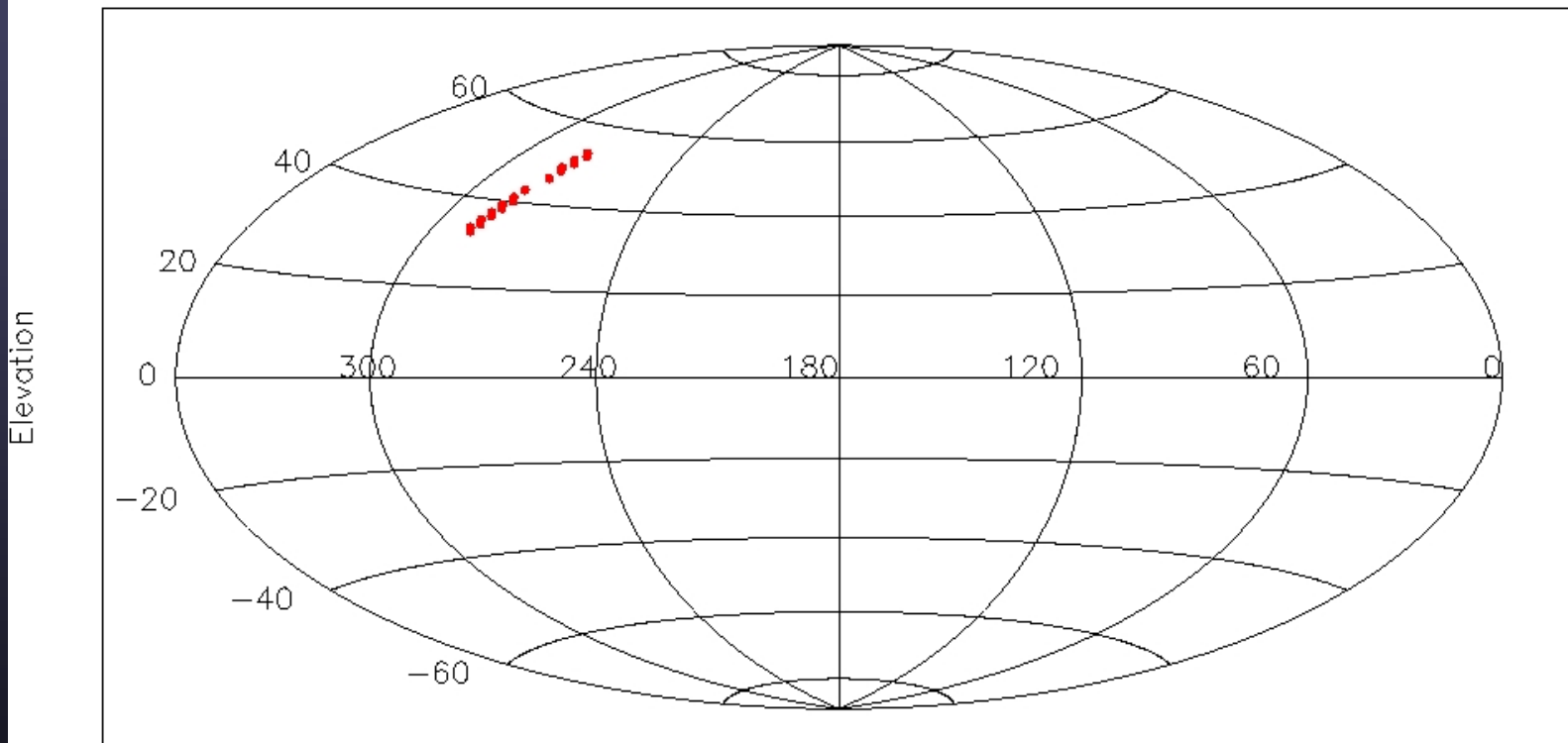
$$\sigma[\Delta f] = 2.1 \text{ mm}$$



Performance – Offset Pointing

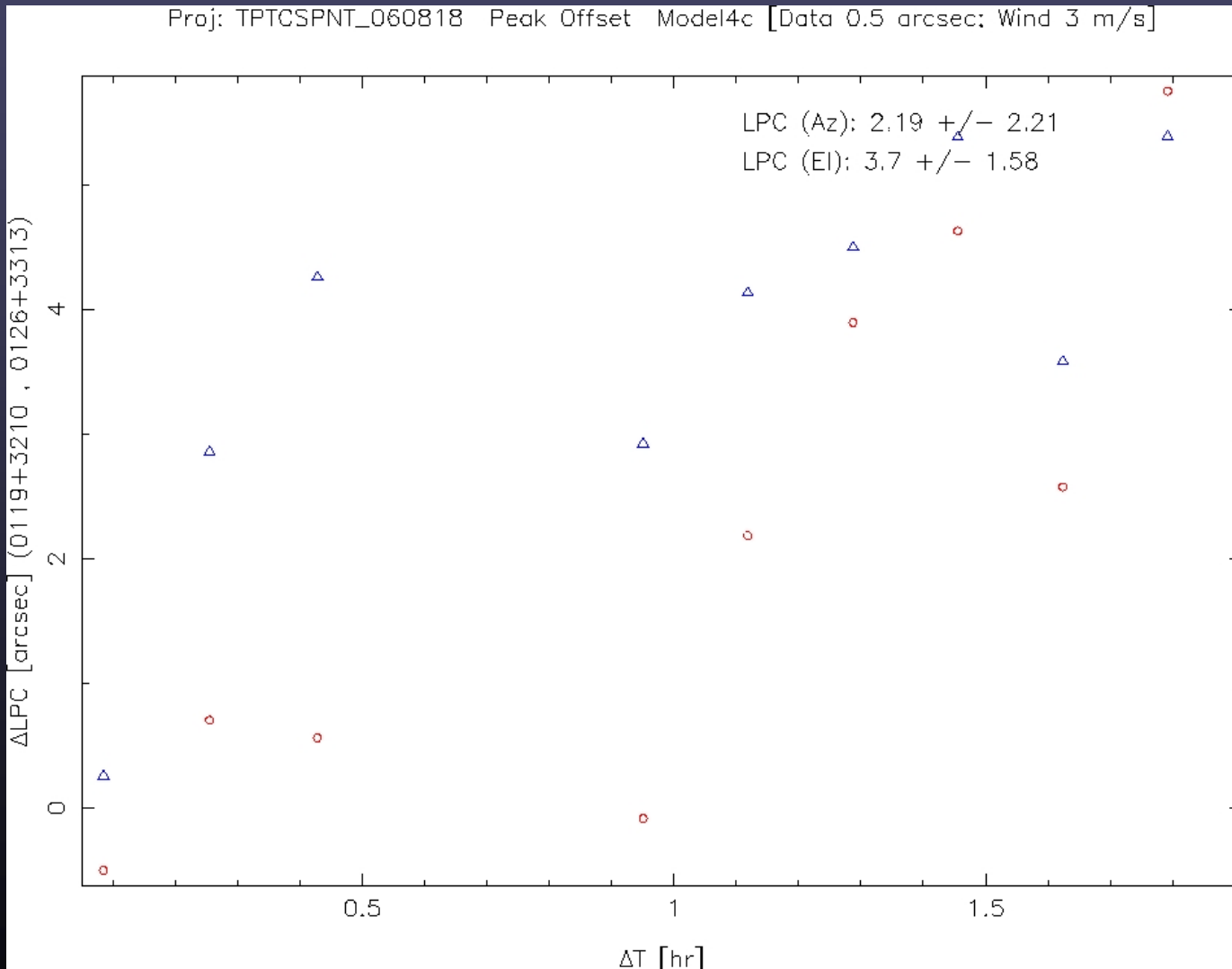


Proj: TPTCSPNT_060818 Peak Model4c [Data 0.5 arcsec; Wind 3 m/s]



Azimuth centered on 180

Offset Pointing Uncertainty

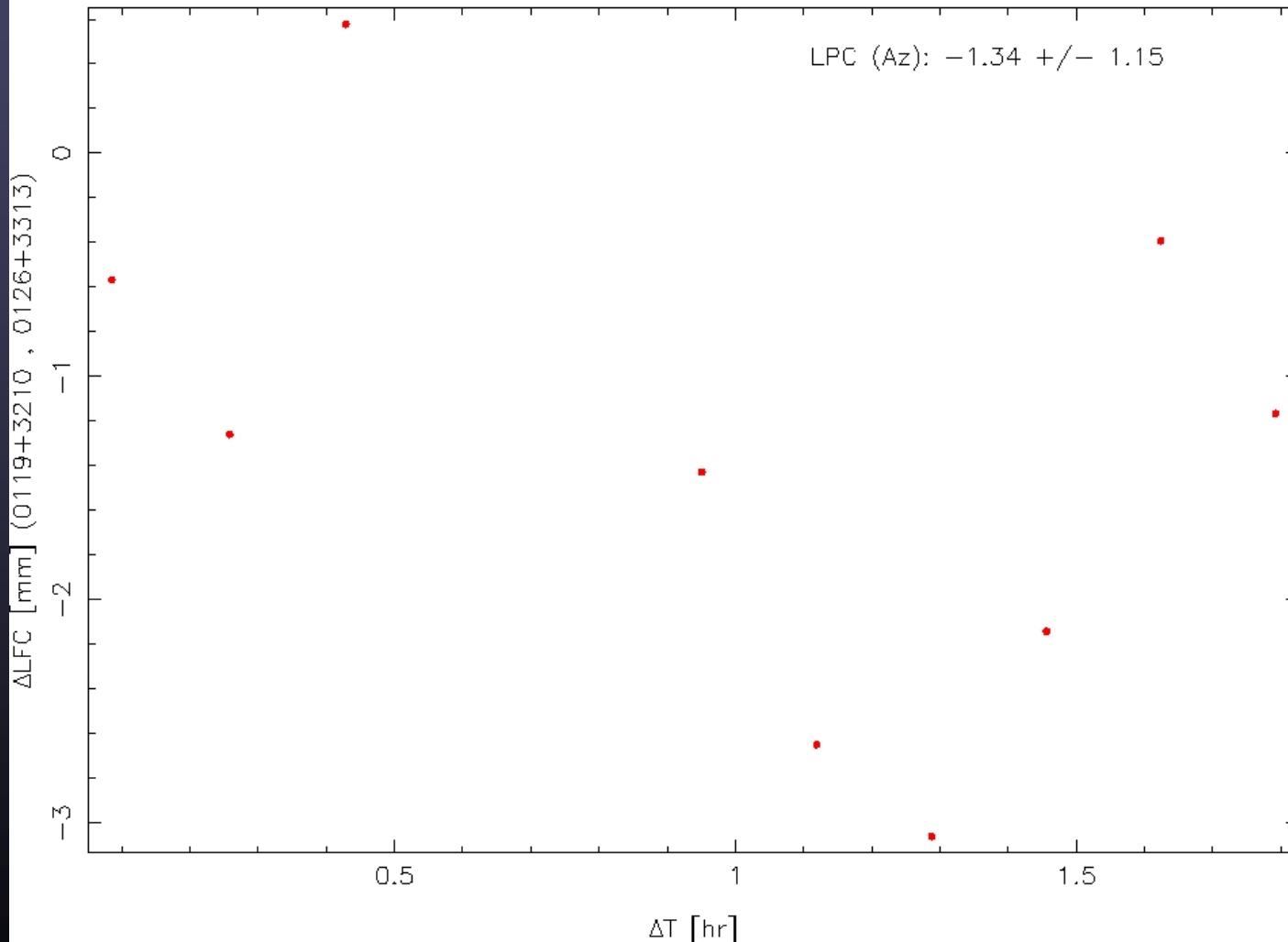


$\sigma_2 \approx 2.7$ arcsec



Offset Focus Uncertainty

Proj: TPTCSPNT_060818 Focus Offset Model4c [Data 0.5 mm; Wind 3 m/s]

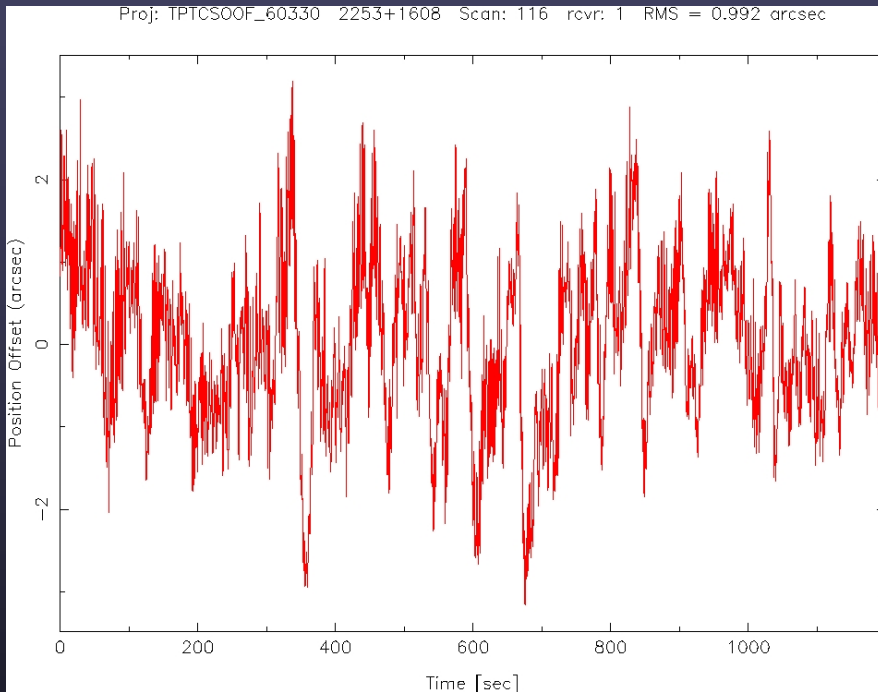


$\sigma \approx 1.2 \text{ mm}$

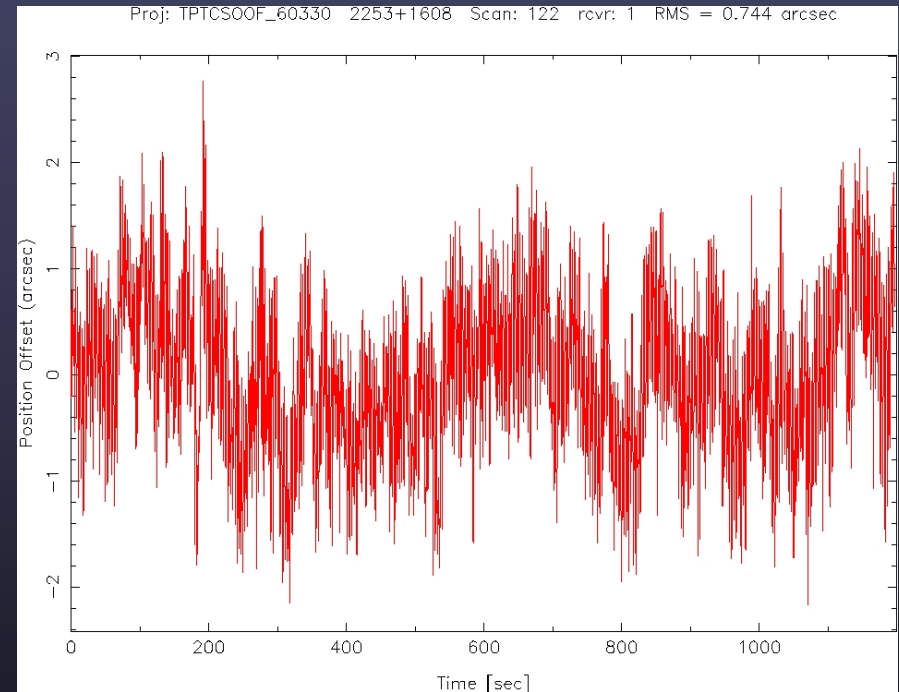
Performance – Tracking



Half-power in Azimuth



Half-power in Elevation



$$(Az, El) \approx (99^\circ, 37^\circ)$$

$$(\Delta Az, \Delta El) \approx (3.6^\circ, 3.9^\circ)$$

$$\left(\frac{\Delta Az}{\Delta t}, \frac{\Delta El}{\Delta t} \right) \approx (10.8' / m, 11.7' / m)$$

$$\sigma_2 \approx 1.2 \text{ arcsec}$$

$$(Az, El) \approx (105^\circ, 43^\circ)$$

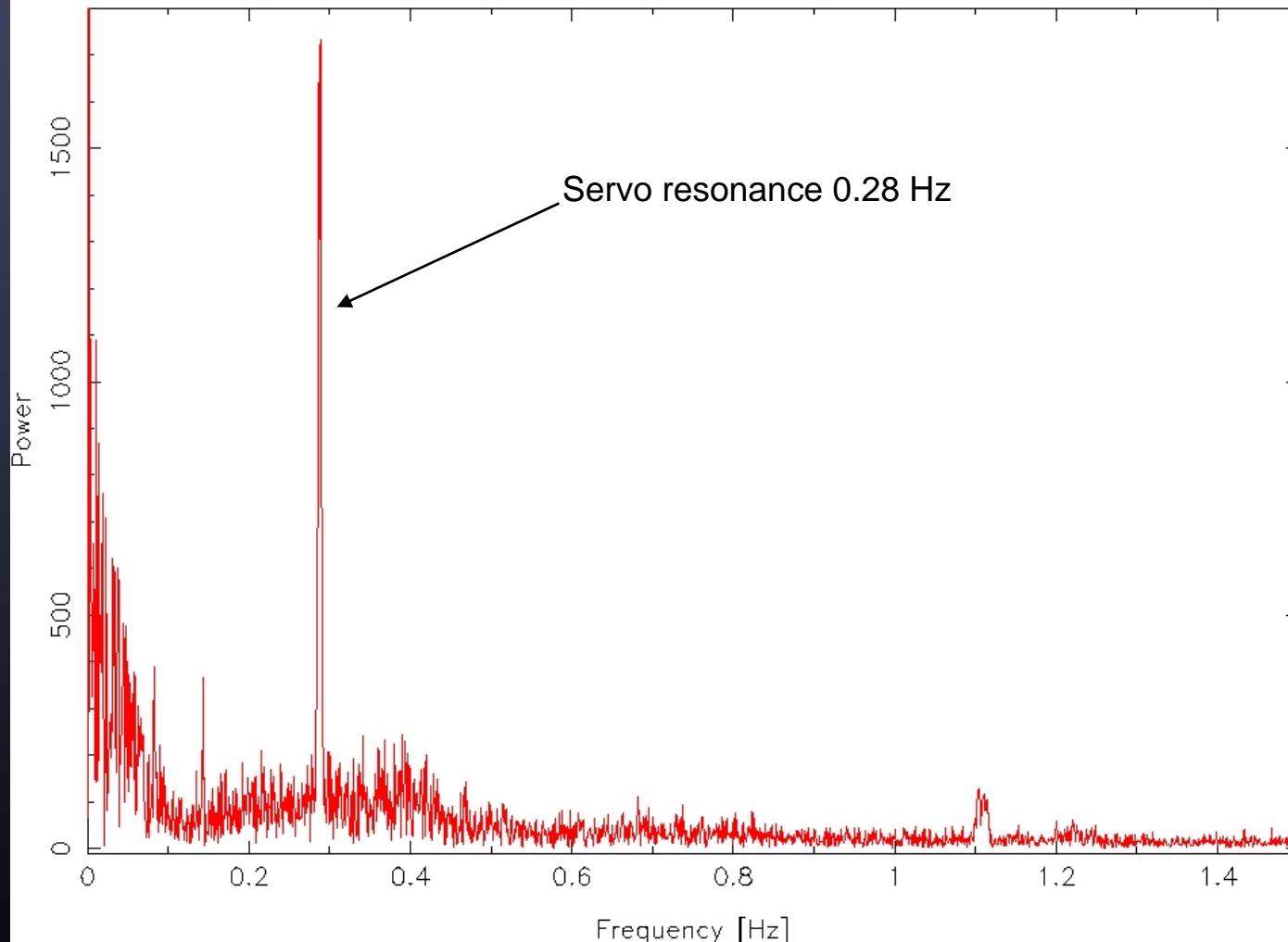
$$(\Delta Az, \Delta El) \approx (4.0^\circ, 3.8^\circ)$$

$$\left(\frac{\Delta Az}{\Delta t}, \frac{\Delta El}{\Delta t} \right) \approx (12.0' / m, 11.4' / m)$$

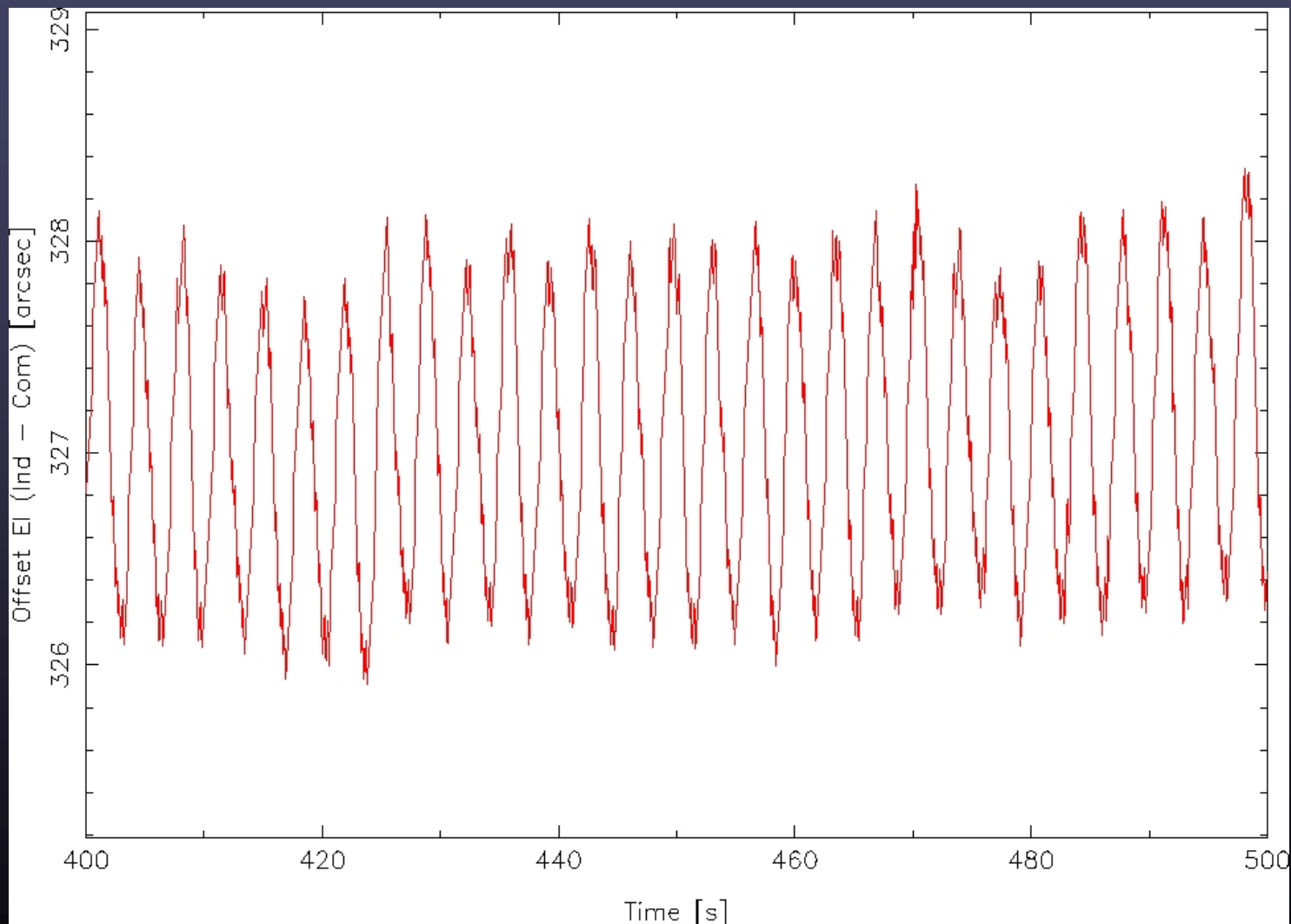
Power Spectrum



Proj: TPTCS00F_60330 2253+1608 Scan: 122 rcvr: 1 RMS = 0.744 arcsec



Servo Error





Performance – Summary

Benign Conditions: (1) Exclude 10:00 → 18:00
(2) Wind < 3.0 m/s

Blind Pointing:
(1 point/focus)

$$\sigma_1(\text{pointing}) \approx 5 \text{ arcsec}$$
$$\sigma(\text{focus}) \approx 2.5 \text{ mm}$$

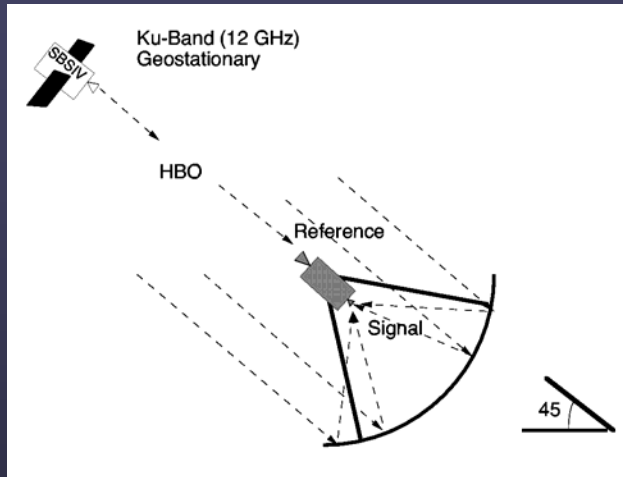
Offset Pointing:
(90 min)

$$\sigma_2(\text{pointing}) \approx 2.7 \text{ arcsec}$$
$$\sigma(\text{focus}) \approx 1.5 \text{ mm}$$

Continuous Tracking:
(30 min)

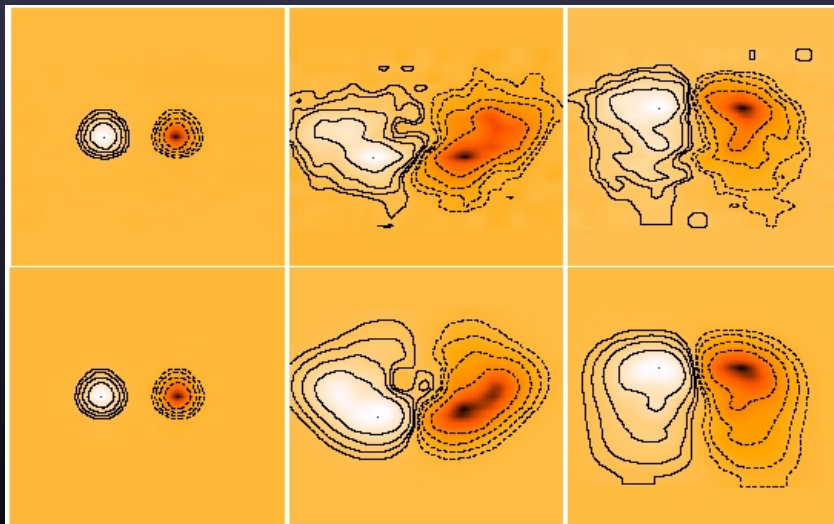
$$\sigma_2 \approx 1 \text{ arcsec}$$

Surface Improvements – Holography



Traditional Holography (Phase Coherent Holography)

Maddalena



Out-of-Focus Holography (Phase Retrieval Holography)

Nikolic et al. (2006)



Zernike Polynomials – Radial Terms

ZERNIKE RADIAL POLYNOMIALS			
n	m	No.	Polynomial
0	0	0	1
1	1	1	$\rho \cos \theta'$
		2	$\rho \sin \theta'$
	0	3	$2\rho^2 - 1$
2	2	4	$\rho^2 \cos 2\theta'$
		5	$\rho^2 \sin 2\theta'$
	1	6	$(3\rho^2 - 2)\rho \cos \theta'$
		7	$(3\rho^2 - 2)\rho \sin \theta'$
	0	8	$6\rho^4 - 6\rho^2 + 1$
	3	3	9
10			$\rho^3 \sin 3\theta'$
2		11	$(4\rho^2 - 3)\rho^2 \cos 2\theta'$
		12	$(4\rho^2 - 3)\rho^2 \sin 2\theta'$
1		13	$(10\rho^4 - 12\rho^2 + 3)\rho \cos \theta'$
		14	$(10\rho^4 - 12\rho^2 + 3)\rho \sin \theta'$
0		15	$20\rho^6 - 30\rho^4 + 12\rho^2 - 1$

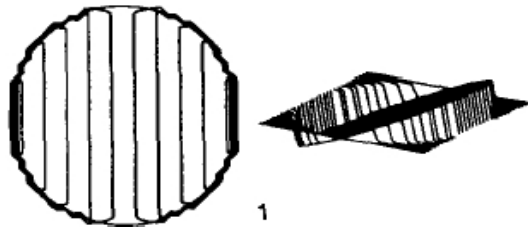
Zernike (1934)
Wyant & Creath (1992)

Zernike Polynomials – Images

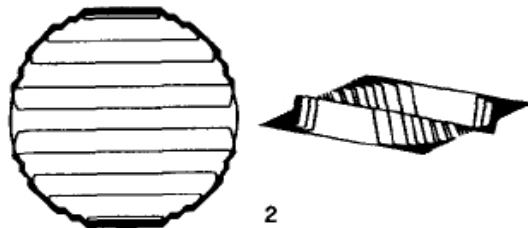


n=1

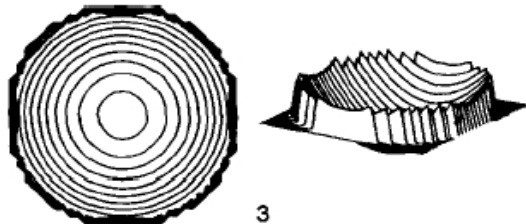
FIRST-ORDER PROPERTIES



TILT



FOCUS



n=2

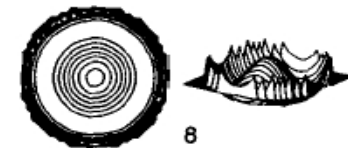
THIRD-ORDER ABERRATIONS



ASTIGMATISM AND DEFOCUS

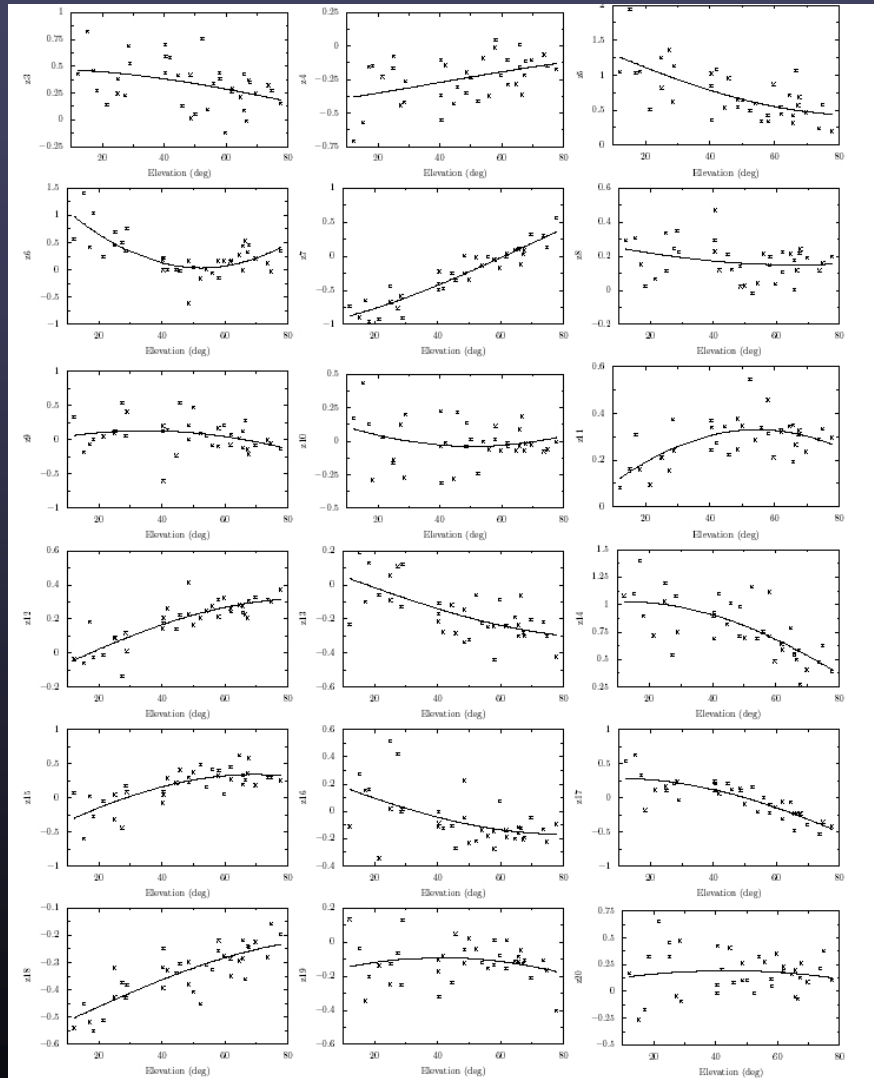


COMA AND TILT



THIRD-ORDER SPHERICAL AND DEFOCUS

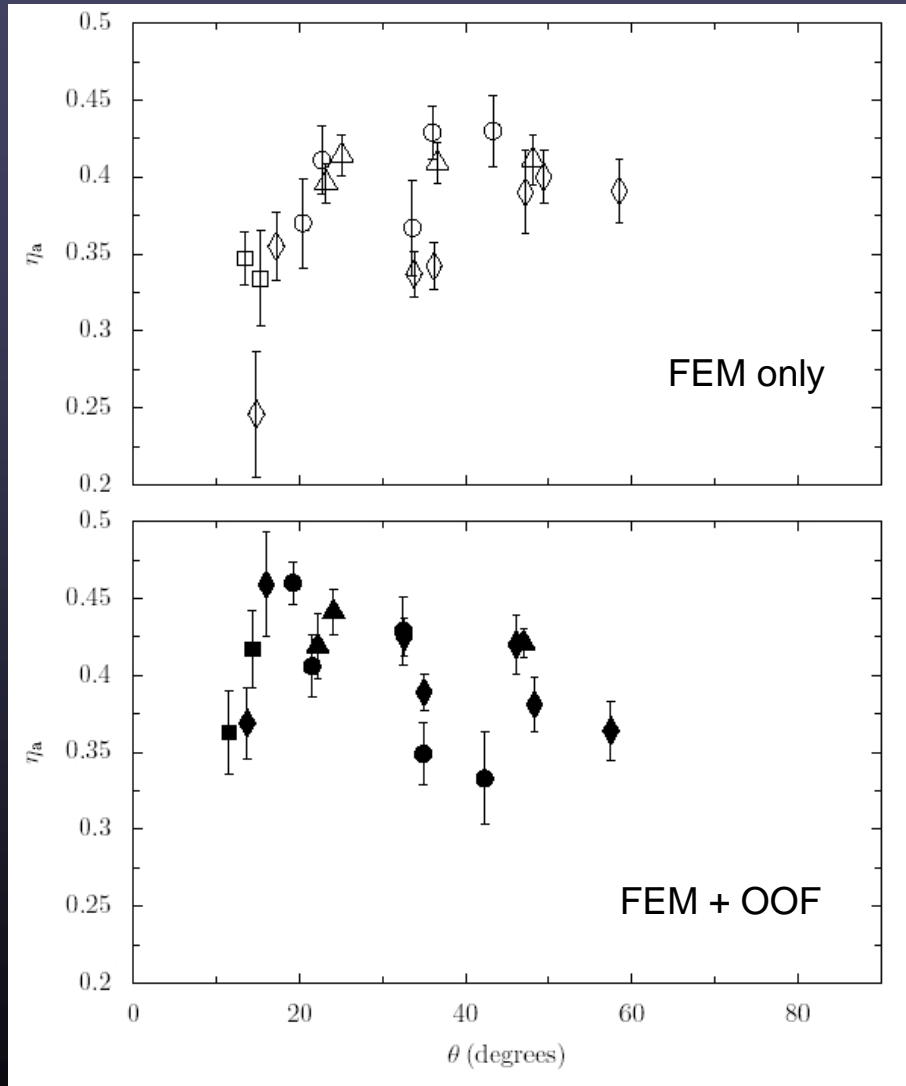
OOF Holography – Gravity



$$z_i(\theta) = a \sin(\theta) + b \cos(\theta) + c$$

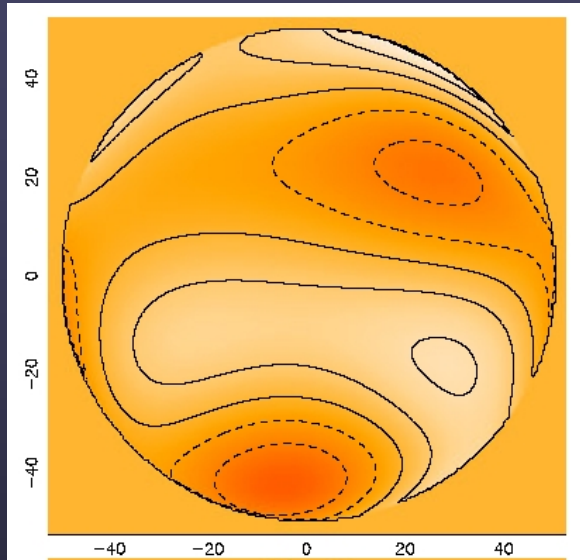
Nikolic et al. (2006)

Gain-Elevation Curve

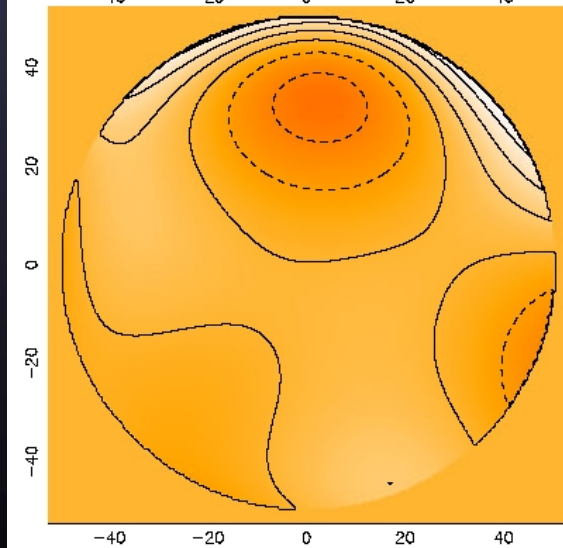


Nikolic et al. (2006)

OOF Holography – Thermal Effects



rms $\approx 330 \mu\text{m}$



rms $\approx 220 \mu\text{m}$

