Memorandum

To: File

From: J. Effland

Date: 2002-01-18

Rev:

2002-12-18	jee	Fixed band figure reference, enlarged font, changed location to CVFILER
2002-01-28	jee	Corrected Figure 1 based on comments from A. R. Kerr.
2002-01-24	jee	Changed G_{IFx} to G_{IF} in Eq. [10] and Eq. [11] based on comments from A. R. Kerr.
		Added reverse bias diagram to Figure 1

Subject: Measurement of Equivalent Noise Temperatures from Mixer Local Oscillators

Equations for calculating the isolation and the equivalent noise temperature from a mixer's local oscillator port are derived here.

Assuming normal bias for the balanced mixer, which means each component-mixer has the opposite polarity, the noise powers at the output of the receiver when the input is connected to a hot and cold load is:

$$P_h = kB\left[G_r\left(T_h + T_r\right) + G_L T_{LO}\right] \qquad\qquad \text{Eq. [1]}$$

$$P_c = kB\left[G_r\left(T_c + T_r\right) + G_L T_{LO}\right] \qquad\qquad \text{Eq. [2]}$$

where:

P_h is the noise power output from the receiver when its input is connected to the hot load
P_c is the noise power output from the receiver when its input is connected to the cold load
G_r is the gain through the receiver system, referenced to the mixer's RF input
G_L is the gain through the receiver system, referenced to the mixer's LO input
T_r is the receiver noise temperature (K)
T_{LO} is the equivalent noise temperature of the noise from the LO (K)
T_h is the hot load's physical temperature (K)
T_c is the cold load's physical temperature (K)
B   is the measurement's noise bandwidth (Hz)
k   is Boltzmann's constant (1.38×10^{-23} W/K/Hz).

Signal paths and gain definitions are diagrammed in Figure 1.

When the bias is reversed for the balanced mixer, which means each component mixer's bias voltage is the same polarity, the noise power paths swap, and the receivers output power when its input is connected to a hot (P_{hx}) and cold load (P_{cx}) is:

$$P_{hx} = kB\left[G_{rx}T_{LO} + G_{Lx}\left(T_h + T_r\right)\right]$$
Eq. [3]

$$P_{cx} = kB\left[G_{rx}T_{LO} + G_{Lx}\left(T_c + T_r\right)\right]$$
Eq. [4]

These equations assume that the receiver's noise temperature and the physical temperature of the hot and cold loads remain the same as when the mixer is normally biased. But gain of the IF system can change between normal and reverse bias conditions because the measurement software adjusts an IF attenuator to maintain a single range on the power meter when measuring hot and cold load noise powers. To account for this gain change, all of the gains and powers are referred to the output of the mixer using the following equations. For normal bias:

$$P_{h,c}^m = \frac{P_{h,c}}{G_{IF}}$$
Eq. [5]

$$G_r^m = \frac{G_r}{G_{IF}}$$
Eq. [6]

and for reverse bias:

$$P_{hx,cx}^m = \frac{P_{hx,cx}}{G_{IFx}}$$
Eq. [7]

$$G_{rx}^m = \frac{G_{rx}}{G_{IFx}}$$
Eq. [8]

where

   $P_{h,c}^m$ are the hot load and cold load powers referred to the mixer output when the mixer is normally biased.

   $P_{hx,cx}^m$ are the hot load and cold load powers referred to the mixer output when the mixer is reverse biased.

   G_r^m is the gain from the mixer's RF port to its output when the mixer is normally biased, and

   G_{rx}^m is the gain from the mixer's LO port to its output when the mixer is reversed biased, and

   G_{IF} is the IF system gain when the mixer is normally biased, and

   G_{IFx} is the IF system gain when the mixer is reversed biased.

The IF gains are found from measured noise powers when the input to the IF system is connected to a hot and cold load using the usual ΔP/ΔT equation:

$$G_{IF} = \frac{1}{kB}\left(\frac{P_{hIF} - P_{cIF}}{T_{hIF} - T_{cIF}}\right)$$
Eq. [9]

where

G_{IF} is the IF system gain when the mixer is normally biased, and
P_{hIF} is the power output from the IF system when its input is connected to a hot load,
P_{cIF} is the power output from the IF system when its input is connected to a cold load,
T_{hIF} is the physical temperature of the hot load connected to the IF input,
T_{cIF} is the physical temperature of the cold load connected to the IF input,

A completely analogous equation is used when the mixer is reversed biased.

Substituting the gain equations in Eq. [5] through Eq. [8] into Eq. [1] and Eq. [2] and then the same equations into Eq. [4] and Eq. [3] gives:

$$P_h = kB\left[G_r^m G_{IF}(T_h + T_r) + G_L^m G_{IF} T_{LO}\right]$$
Eq. [10]

$$P_c = kB\left[G_r^m G_{IF}(T_c + T_r) + G_L^m G_{IF} T_{LO}\right]$$
Eq. [11]

$$P_{hx} = kB\left[G_r^m G_{IFx} T_{LO} + G_L^m G_{IFx}(T_h + T_r)\right]$$
Eq. [12]

$$P_{cx} = kB\left[G_r^m G_{IFx} T_{LO} + G_L^m G_{IFx}(T_c + T_r)\right]$$
Eq. [13]

The gains of each path are found by subtracting Eq. [11] from Eq. [10]:

$$G_r^m = \frac{1}{kBG_{IF}}\left(\frac{P_h - P_c}{T_h - T_c}\right)$$
Eq. [14]

and similarly, subtracting Eq. [13] from Eq. [12] gives:

$$G_L^m = \frac{1}{kBG_{IFx}}\left(\frac{P_{hx} - P_{cx}}{T_h - T_c}\right)$$
Eq. [15]

From Eq. [10]

$$T_h + T_r = \frac{\dfrac{P_h}{kB} - G_L^m G_{IF} T_{LO}}{G_r^m G_{IF}} \qquad\qquad \text{Eq. [16]}$$

and substituting this into Eq. [12] gives:

$$
\begin{aligned}
P_{hx} &= kB\left[G_r^m G_{IFx} T_{LO} + G_L^m G_{IFx}\left(\frac{\dfrac{P_h}{kB} - G_L^m G_{IF} T_{LO}}{G_r^m G_{IF}} \right) \right] \\[2mm]
&= kB G_r^m G_{IFx} T_{LO} + \frac{G_L^m G_{IFx}}{G_r^m G_{IF}} P_h - kB \frac{\left(G_L^m\right)^2 G_{IFx}}{G_r^m} T_{LO} \\[2mm]
&= kB G_r^m G_{IFx}\left[1 - \left(\frac{G_L^m}{G_r^m}\right)^2 \right] T_{LO} + \frac{G_L^m G_{IFx}}{G_r^m G_{IF}} P_h
\end{aligned}
\qquad \text{Eq. [17]}
$$

Solving this for T_{LO}:

$$T_{LO} = \frac{P_{hx} - \dfrac{G_L^m G_{IFx}}{G_r^m G_{IF}} P_h}{kB G_r^m G_{IFx}\left[1 - \left(\dfrac{G_L^m}{G_r^m}\right)^2 \right]} \qquad\qquad \text{Eq. [18]}$$

If the ratio of the mixer's LO-to-IF and RF-to-IF port gains are defined as the LO isolation $\left(I = \dfrac{G_L^m}{G_r^m} \right)$, then Eq. [18] can be slightly simplified to

$$\boxed{\; T_{LO} = \frac{P_{hx} - \left(\dfrac{G_{IFx}}{G_{IF}}\right) I\, P_h}{kB G_r^m G_{IFx}\left(1 - I^2\right)} \;} \qquad\qquad \text{Eq. [19]}$$

As an alternative, we can also derive an equation for T_{LO} by using the cold load powers and temperatures of Eq. [11] to create an analog to Eq. [16] which can be substituted into Eq. [13] to yield:

$$T_{LO} = \frac{P_{cx} - \left(\dfrac{G_{IFx}}{G_{IF}}\right) I\, P_c}{kB G_r^m G_{IFx}\left(1 - I^2\right)} \qquad\qquad \text{Eq. [20]}$$

This is the LO noise referred to the LO port of the mixer. To refer it to the mixer's RF input, use the isolation:

$$T_{LO@Input} = I\ T_{LO}$$ Eq. [21]

Check

Table 1 shows the derivation of the values given actual powers and temperatures from balanced mixer/preamp UVaVIII-L811B-1-B2-3-GL-BM371-A+BM1F4-12P.01-A. The measured data is for an LO frequency of 260 GHz and an IF of 6.67 GHz.

Receiver System Measurements		
P_h	-19.94 dBm	1.01E-05 W
P_c	-23.64 dBm	4.33E-06 W
P_{hx}	-20.35 dBm	9.23E-06 W
P_{cx}	-23.05 dBm	4.95E-06 W
T_h	300.1 K	
T_c	93.5 K	
IF System Measurements		
T_h	48.9 K	
T_c	4.3 K	
T_{hx}	49.1 K	
T_{cx}	4.3 K	
P_h	-44.59 dBm	3.47E-08 W
P_c	-49.76 dBm	1.06E-08 W
P_{hx}	-41.83 dBm	6.57E-08 W
P_{cx}	-46.94 dBm	2.02E-08 W
Calculations		
Y_{rf}	3.70 dB	2.344
T_r	60.2 K	
Y_{if}	5.16 dB	3.282
T_{if}	15.2 K	
kBG_{IF}	5.42E-10 W/K	
kBG_{IFx}	1.02E-09 W/K	
kBG_r^m	5.19E+01 W/K	
kBG_L^m	2.04E+01 W/K	
I	3.92E-01	-4.07 dB
T_{lo}	39.8 K	
$T_{lo@Input}$	15.6 K	

Table 1 : Calculation Check
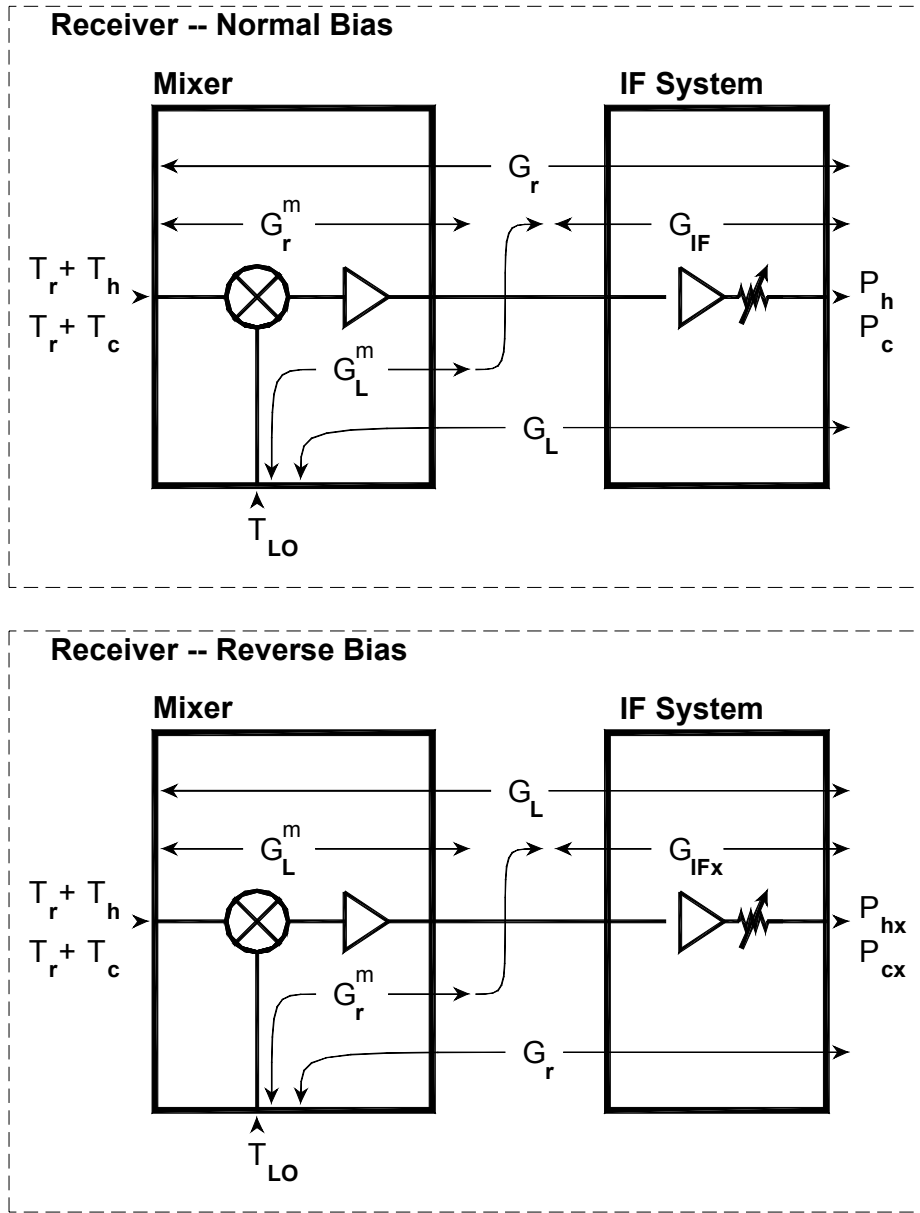
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Figure 1: Gain Definitions