Memorandum

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cc: J. Webber

From: J. Effland

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Subject: Error Sensitivities for Noise Temperature Measurements

Summary

This memo presents the sensitivity of noise measurements to changes in the physical temperatures of the hot and cold loads or to changes in the measured Y-Factor. The results are presented as percent error, which is the change in a parameter divided by the nominal value of the parameter. The included graph shows the percent error of the receiver noise temperature that results when the hot load, cold load, or Y Factor changes by 1%. Motivation for this analysis resulted from recent repeatability problems and discrepancies between the automated chopper-based measurement system and hand-held conical hot and cold loads.

In summary, the fractional errors for all three measurement parameters increase with decreasing receiver noise temperature, and the chopper-based measurement system increases error sensitivity by about 25%. The chopper-based measurement system degrades error sensitivity because the effective noise temperature of the cold load used with that system is 93.5K compared to 78K for the conical load.

Errors from Changes in Hot Load

The equation for receiver noise temperature (T_r) using the Y-Factor technique measures the ratio of noise powers output from the receiver while its input is connected to a hot load of physical temperature T_h and then connected to a cold load of physical temperature T_c:

$$Y = \frac{T_r + T_h}{T_r + T_c} \qquad \text{(Eq. 1)}$$

The receiver noise temperature is obtained by solving Eq. 1 for T_r is:

$$T_r = \frac{T_h - Y\,T_c}{Y - 1} \qquad \text{(Eq. 2)}$$

The fractional error, or error sensitivity, in receiver noise temperature as a function of the fractional error in the physical temperature of the hot load, is obtained by differentiating Eq. 2 with respect to the hot load temperature, multiplying each side of the equation by $1/(T_h\,T_r)$, and rearranging:

$$\frac{\partial T_r}{T_r} = \left(1 - Y\frac{T_c}{T_h}\right)^{-1} \frac{\partial T_h}{T_h} \qquad \text{(Eq. 3)}$$

The top two curves in Figure 1 show the fractional error in receiver noise temperature when the error in hot load temperature is 1%. The solid curve shows the error when conical loads are used, and the dashed curve shows error sensitivities when using the chopper-based loads.

The effective noise temperature of the chopper based cold load, which is 7% higher than the conical load, increases the error sensitivity when measuring receivers with low noise temperatures. As an example, Figure 1 shows that when measuring a 40K receiver the sensitivity to hot load errors increases from about 4% with the conical cold load to nearly 5% when using the chopper-based loads.

Errors from Changes in Cold Load

Errors in the cold load physical temperature produce sensitivity errors in an analogous manner:

$$\frac{\partial T_r}{T_r} = \left(1 - \frac{1}{Y\frac{T_c}{T_h}}\right)^{-1} \frac{\partial T_c}{T_c} \qquad \text{(Eq. 4)}$$

The curves shown in Figure 1 for this case are similar to the error with hot load temperature, but their sign is opposite. This means that measurement error from the hot and cold load temperature errors would nearly cancel if the hot and cold load temperature errors tracked each other.

Errors from Changes in Y-Factor

Errors in receiver noise temperature from Y-Factor errors is derived by first differentiating Eq. 2 with respect to Y and then rearranging the results into a fractional form:

$$\begin{aligned}
\frac{\partial T_r}{\partial Y} &= \frac{-T_h}{(Y-1)^2} - \frac{\partial}{\partial Y}\left(\frac{YT_c}{Y-1}\right) \\
&= \frac{-T_h}{(Y-1)^2} + \left(\frac{-T_c}{Y-1} + \frac{YT_c}{(Y-1)^2}\right) \qquad \text{(Eq. 5)} \\
&= \frac{-T_h + YT_c - YT_c + T_c}{(Y-1)^2} \\
&= \frac{T_c - T_h}{(Y-1)^2}
\end{aligned}$$

The fractional error in receiver temperature with respect to the Y-Factor flows immediately from Eq. 5 by multiplying both sides by Y/T_r and rearranging to obtain:

$$\frac{\partial T_r}{T_r} = \frac{\left(T_c - T_h\right)Y}{\left(Y-1\right)^2 T_r}\frac{\partial Y}{Y} \qquad \text{(Eq. 6)}$$

Unlike the hot and cold load errors, which monotonically increase with decreasing receiver temperature, Y-Factor errors are magnified when measuring either high or low temperature receivers as graphed in Figure 1. This is expected because high receiver noise temperatures yield small Y-Factors so significant errors in receiver noise temperature result from small errors in the Y Factor. For low receiver temperatures, the Y-Factor is large, but the small difference (T_h - YT_c) in the numerator of Eq. 2 is then sensitive to changes in Y.

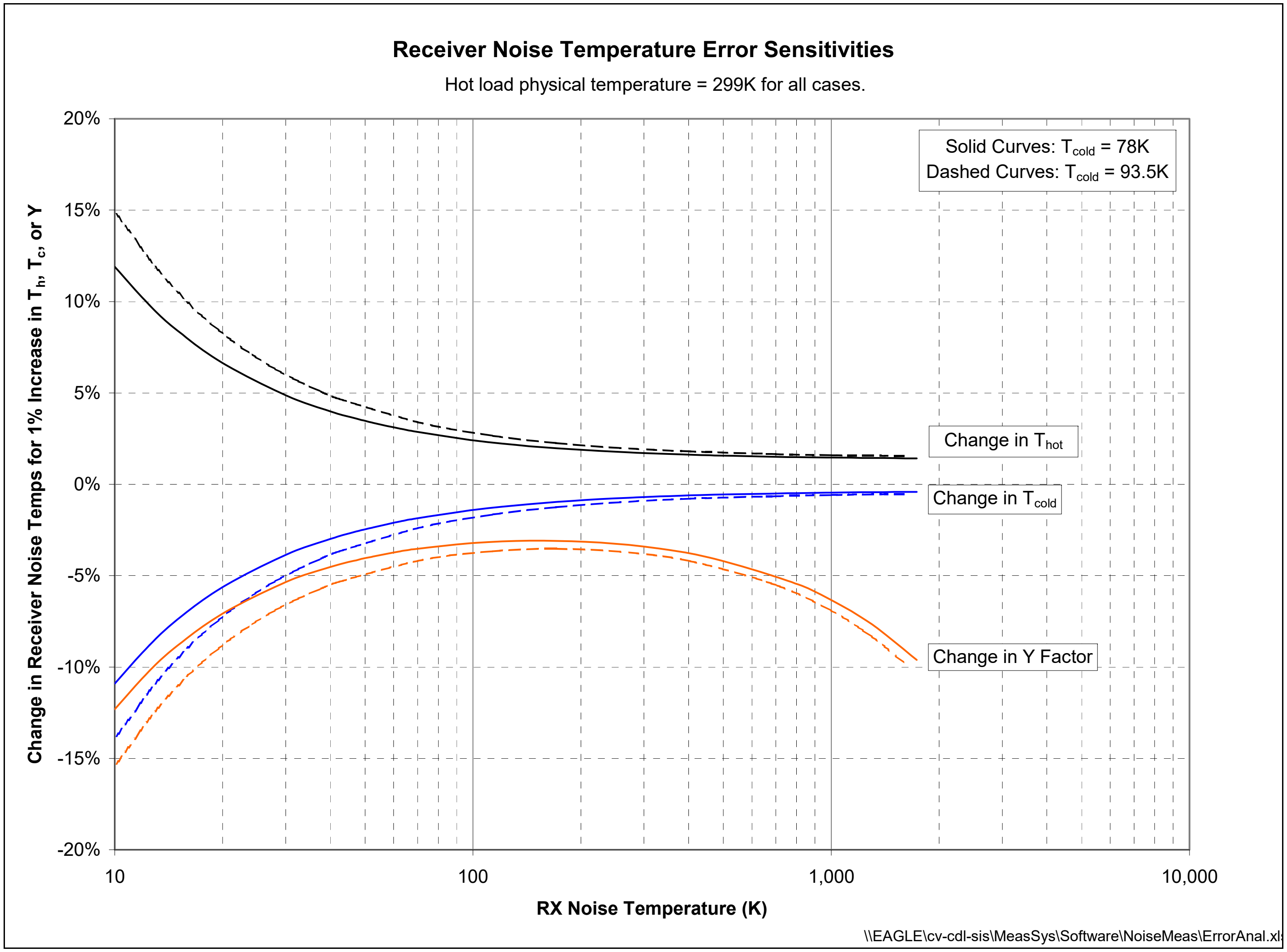

Figure 1: Receiver Noise Temperature Error Sensitivities