GBT LO and Doppler Corrections

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1 Introduction

This note outlines the methods by which GBT observers can specify observing frequencies or velocities and the equations that are used for Doppler corrections as they affect the setting of the first local oscillator(s). Comments will be appreciated.

Part of the translation of user commands to first local oscillator commands are handled in the "GBT Observe" observerer interface and part in M&C's LO1 manager software. The interface between the two is a sky frequency track for the duration of the scan in one of the selected inertial rest frames. GBT Observe translates velocities specified by the observer into sky frequencies, and the LO1 manager applies Doppler corrections from the selected rest frame to the rest frame of the GBT. The observer may also specify sky frequencies directly in any of the recognized rest frames.

2 Rest Frames

Five inertial rest frames for the purpose of Doppler corrections are recognized by the GBT: local (no Doppler corrections), the sun (heliocentric), the local standard of rest (LSR) as defined by nearby stars, the center of the Milky Way, and the inertial frame defined by the cosmic microwave background. These rest frames are implemented primarily as an observing convenience. The data are recorded with the frequencies of the GBT local oscillators so the task of interpreting frequencies as velocities is left to the data reduction software.

The heliocentric rest frame is define by the latest JPL Solar System Ephemeris, currently DE405. This frame is commonly used for published velocities of extragalactic objects. The difference between the local and heliocentric rest frames is approximately 30 km/s, mostly due to the earth's rotation around the sun. The GBT's velocity with respect to the center of the earth is about 0.4 km/s. The difference between the heliocentric and solar system barycenter rest frame is less than 0.02 km/s.

The literature differs on the definition the local standard of rest. The GBT has adopted the convention in use since around 1960 which assigns the velocity of the sun a value of 20 km/s in the direction $\alpha = 18^h$, $\delta = +30^o$ (1900), which precesses to 18:03:50.27, +30:00:16.8 (J2000). (See, for example, Kerr & Lynden-Bell, 1986, MNRAS, vol. 221, p1023.) Other solar motion vectors found in print include 19.5 km/s toward 18:02:36, +29:56:41 (J2000) ($l = 56^o, b = +23^o$), 15.4 km/s toward 17:55:36, +25:36:38 (J2000) ($l = 51^o, b = +23^o$), and 16.5 km/s toward 17:49:53, +28:00:02 (J2000) ($l = 53^o, b = +25^o$) all found in Delhaye, 1965, Stars and Stellar Systems, Vol. 5, Galactic Structure, ed. Blaauw & Schmidt, Univ. of Chicago Press, p73-4. Check your source of LSR velocities to be sure of the rest frame definition used.

The Galactocentric rest frame was defined indirectly by the IAU in 1985 when the standard value for the velocity of the local standard of rest was adopted to be 220 km/s in the direction $l = 90^o, b = 0^o$ (α = 21:12:01.05, δ = +48:19:46.7 (J2000)). Combining this with the solar motion vector with respect to the LSR adopted above give a solar vector in the Galactocentric rest frame of 235.669 km/s toward 20:53:40.66, +47:42:38.6 (J2000).

The inertial frame tied to the cosmic background radiation is presently defined by the COBE satellite experiment (Kogut, et. al., 1993, Astrophys. J., vol. 419, p1). The solar motion vector is 369.5 km/s in the direction $l = 264.4^o, b = 48.4^o$ (α = 11:12:56.43, δ = -06:57:50.0 (J2000)).

3 Velocity Definitions

The GBT's observer interface recognizes four definitions of redshift from which the receiver's center frequency may be determined. Two are pseudo-velocities, commonly called "radio" and "optical" velocities, the third is a closer approximation to a real spatial velocity, which we call "relativistic," and the fourth is the standard definition of redshift, "z."

Radio velocity is defined by the equation

$$V_{rad} = \frac{f_0 - f}{f_0} c = \frac{\lambda - \lambda_0}{\lambda} c, \tag{1}$$

where f_0 and λ_0 are the rest frequency and wavelength of the observed spectral line, and f and λ are the measured frequency and wavelength, respectively, and c is the speed of light. The understanding is that V_{rad} is the line-of-sight component of the relative velocity of the observer and observed object.

Optical velocity is defined by the equation

$$V_{opt} = \frac{f_0 - f}{f} c = \frac{\lambda - \lambda_0}{\lambda_0} c. \tag{2}$$

Again, V_{opt} is understood to be the line-of-sight velocity component.

The differences between radio and optical velocities are 0.04, 3.3, and 345 km/s at redshifts of 100, 1000, and 10,000 km/s, respectively. Most velocities published for Galactic objects use the radio definition, and nearly all extragalactic velocities use the optical definition.

A more physically meaningful relationship between observed frequency and relative observer-object velocity is described by special relativity in the equation

$$f = \frac{f_0 \sqrt{1.0 - (\frac{V}{c})^2}}{1.0 + \frac{V cos\alpha}{c}}, \tag{3}$$

where V is the three-dimensional observer-object velocity difference, and α is the angle between the line of sight and the relative velocity vector. Since α and, hence, the three-dimensional velocity, V, are seldom known, and one $\frac{V}{c}$term in Equation **??** does not depend on $cos\alpha$, we cannot solve even for the line-of-sight component of the velocity. However, if we assume that $\alpha = 0$ and solve for V we get a closer approximation to the radial component of the relative kinematic velocity,

$$V_{rel} = \frac{f_0^2 - f^2}{f_0^2 + f^2} c \tag{4}$$

Very few, if any redshifts have ever been published with this velocity definition. If the 1420.4058 MHz line of HI is measured at a frequency of 1373.026 MHz, the three velocity definition values are $V_{rad} = 10000$ km/s, $V_{opt} = 10345$ km/s, and $V_{rel} = 10167$ km/s.

Large redshifts are normally published as the dimensionless quantity

$$z = \frac{f_0 - f}{f}. \tag{5}$$

This is the optical definition without the speed-of-light multiplier.

When interpreting spectral line widths or setting spectrometer bandwidths at high redshift keep in mind that dV/df differs markedly for the three velocity definitions. At $z = 1$, $dV_{rad}/df = c/f_0$, $dV_{opt}/df = 4cf_0$, and $dV_{rel}/df = 1.28cf_0$. The only useful definition in this case is the relativistic one:

$$\frac{dV_{rel}}{df} = \frac{-4cff_0^2}{(f^2 + f_0^2)^2} \tag{6}$$

and

$$\frac{df}{dV} = \frac{-f_0}{(c + V_{rel})\sqrt{1 - (\frac{V_{rel}}{c})^2}} \tag{7}$$

4 Doppler Corrections

In a sense, the correction for Doppler shift in setting the GBT's first LO is done in two steps. The first step is simply a reversal of the interpretation of velocity from an observed spectral line frequency as described in Section **??**. The second step is a true Doppler correction from the specified rest frame described in Section **??** to the local rest frame. Equation **??** is the best one for this purpose, where f_0 is now the spectral line frequency in the chosen rest frame.

It might be interesting to note here that relativistic velocities and Doppler shifts add correctly when using an intermediate rest frame, while velocities derived from the other three definitions in Section **??** do not. We are still left with the problem of not knowing the true three-dimensional space velocity of the observed object, but for rest frame corrections recognized by the GBT this is a quite small second order error. In one dimension the relativistic addition of velocities is given by

$$V = \frac{V_1 + V_2}{1 + \frac{V_1 V_2}{c^2}} \tag{8}$$

(See, for example, Bergman, 1942, Introduction to the Theory of Relativity, Prentice-Hall, Inc., Englewood Cliffs, NJ, p43.) Applying Equation **??** once using V from Equation **??** or twice using V_1 and then V_2 will yield the same Doppler shifted frequency.

5 Solar System Objects

Since solar system objects are at a finite distance when compared to the orbital motion of the earth, velocities must be handled as the addition of three-dimensional vectors for the time of observations. The GBT observer interface provides for automatic computation of Doppler shifts for the major planets using the JPL Solar System Ephemeris and for other objects for which orbital elements can be specified. The observer may also provide a pre-computed frequency ephemeris in the GBT's local rest frame.