Astronomical Calibration
Sources for ALMA

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The specification for ALMA amplitude calibration is:

- 1% accuracy at millimeter wavelengths ($\nu < 600$ GHz);
- 3% accuracy at submillimeter wavelengths ($\nu > 600$ GHz).

**THIS IS PRETTY TOUGH!!!** Consider:

- current mm interferometers only good to 10% at best;
- little experience in submm interferometry;
- even in radio, where things easier (relatively), only good to about 5% or so (slightly better from 1-15 GHz).
Amplitude Calibration
Options

Two possibilities for amplitude calibration:

- **ab initio**
  if all telescope properties are known and/or measured accurately enough, then measured correlation coefficients can be turned directly into calibrated (in amplitude) visibilities.

- **a posteriori**
  observe astronomical sources of “known” flux density and use those observations to calibrate the amplitudes.
The fundamental measured quantity of an interferometer is the \textit{correlation coefficient}. This is turned into a calibrated \textit{visibility} via:

\[
V_{ij} = \rho_{ij} e^{\frac{1}{2}(\tau_i + \tau_j)} \sqrt{G_i T_{\text{sys},i} G_j T_{\text{sys},j}}
\]

where \[
G_i = \frac{2k}{A_i \eta_{a_i}}
\]

So, if the system temperature, aperture efficiency, and opacities are known accurately enough, there is no need to use astronomical sources for \textit{a posteriori} calibration.
Ab Initio Calibration

Problems with \textit{a priori} calibration include:

- need to accurately measure system temperature, aperture efficiency (actually, full 2-D antenna voltage pattern), and atmospheric opacity (at each antenna);
- must accurately set focus, delay, and pointing;
- decorrelation effects must be accounted for.

Benefits are:

- no need for extra observations (scheduling is easier);
- no need to assume you know the flux density of astronomical sources.
A \textit{Posteriori} Calibration

If you cannot know or measure the telescope properties well enough, then you can turn the correlation coefficient into a calibrated (in amplitude) visibility by observing a source of known flux density, and directly determining the conversion factor. The flux density can be known via:

\begin{itemize}
  \item calculation from first principles;
  \item observation with an accurately calibrated telescope;
  \item combination of the above two.
\end{itemize}
A Posteriori Calibration

Problems with a posteriori calibration include:

- difficulty in knowing absolute flux density of sources;
- decorrelation effects must be accounted for;
- must still measure $T_{sys}$ and voltage pattern (relative).

Benefits are:

- $T_{sys}$ and voltage pattern measurements can be relative;
- not necessary to know absolute gain or opacity (unless a correction for different elevation is required).
A Posteriori Calibration

Generally, there are very few sources which are true absolute calibration standards (primary calibration sources). Since there are so few of them, in order to make it possible to find calibrators at more times/elevations, a number of other sources are observed along with the primary sources, and their flux density is bootstrapped from the primary (secondary calibration sources). We would like to have some 10’s of these sources. They must be regularly monitored, along with the true primary calibration sources, as they can vary on even short timescales.
Types of sources which could be (and have been) primary or secondary calibrators:

- extragalactic (QSOs) – e.g., Cygnus A, 3C286;
- HII (or UCHII or HCHII) regions – e.g., W3(OH), DR21;
- stars, at all ages – e.g., Cas A, NGC 7027, MWC 349;
- solar system – e.g., Mars, Jupiter.
How strong do the amplitude calibration sources need to be? We need to have the accuracy of a (relatively short – a few minutes at most) observation have uncertainty dominated by the uncertainty in the flux density of the source itself, not by the uncertainty from the thermal rms. So, the source flux density should be $\geq$ the thermal rms on a single baseline (or so). In fact, we relax this because we know we will use self-calibration, so the appropriate thermal rms is not for a single baseline, but for the entire array. So, use a criteria that the source flux density is $\geq 100 \times$ the thermal rms of the entire array (for 1% accuracy).
# ALMA Sensitivity

<table>
<thead>
<tr>
<th>frequency (GHz)</th>
<th>1-σ in 1 min (mJy)</th>
<th>required flux density (mJy)</th>
</tr>
</thead>
<tbody>
<tr>
<td>35</td>
<td>0.02</td>
<td>2.0</td>
</tr>
<tr>
<td>90</td>
<td>0.03</td>
<td>3.0</td>
</tr>
<tr>
<td>230</td>
<td>0.07</td>
<td>7.0</td>
</tr>
<tr>
<td>350</td>
<td>0.20</td>
<td>20.0</td>
</tr>
<tr>
<td>675</td>
<td>0.70</td>
<td>70.0</td>
</tr>
<tr>
<td>850</td>
<td>1.10</td>
<td>110.0</td>
</tr>
</tbody>
</table>
How large can the amplitude calibration sources be? We generally want the source to be significantly smaller than the resolution of the telescope or interferometer, to avoid problems in either having to know the 2-D voltage pattern of the antennas, or extrapolating to the zero spacing flux density.
# ALMA Resolution

<table>
<thead>
<tr>
<th>frequency (GHz)</th>
<th>antenna resolution (asec)</th>
<th>compact config resolution (asec)</th>
<th>14-km config resolution (masec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>35</td>
<td>150</td>
<td>8.8</td>
<td>130</td>
</tr>
<tr>
<td>90</td>
<td>60</td>
<td>3.4</td>
<td>50</td>
</tr>
<tr>
<td>230</td>
<td>24</td>
<td>1.3</td>
<td>20</td>
</tr>
<tr>
<td>350</td>
<td>15</td>
<td>0.9</td>
<td>13</td>
</tr>
<tr>
<td>675</td>
<td>8</td>
<td>0.5</td>
<td>7</td>
</tr>
<tr>
<td>850</td>
<td>6</td>
<td>0.4</td>
<td>5</td>
</tr>
</tbody>
</table>
Calibration Sources: QSOs

At radio wavelengths, the primary flux density calibrators are mostly external radio galaxies, e.g., at the VLA, the standard is 3C295, which is used to monitor 3C286, 3C48, etc… every 16 months. 3C286 and 3C48 are secondary flux density calibrators (but are effectively used as if they are primaries). Their variations are small and slow, on physical grounds – the emission is dominated by the radio lobes. By the time you get to the mm/submm, the emission is generally weaker, and dominated by the core (lobes go like $\lambda^{0.7}$ while core is closer to flat spectrum), which is variable. So, while they might be good secondaries, these sources are probably not useful as primaries.
Calibration Sources: Stars

Main sequence stars are small in angular size, so are good in that respect, but are too weak (the brightest are of order a few mJy at 650 GHz) to be considered as viable primary or secondary calibrators.

Giant and supergiant stars, however, although cooler, are much larger and hence brighter. The brighter ones have flux density on the order of 10’s of mJy at 650 GHz (and scale mostly like $\lambda^{-2}$). Their sizes are typically a few masec. They therefore might be reasonable candidates for secondary calibrators (but are weak). They are generally too variable to be considered as primary calibrators.
Calibration Sources: HII regions & PN

HII (and UCHII) regions and PN have been used as calibrators in the mm/submm for many years. Such sources include DR21, W3(OH), NGC7027, K3-50A, etc…

They are typically a few Jy, and are hence easily strong enough, however, they are typically too large to consider as primary calibrators for ALMA, with sizes on the order of a few to 10’s of arcseconds. Small UCHIIIs or HCHIIIs might be good candidates for secondaries, but this is a research topic. Most of these sources are variable to some degree, so would have to be monitored. There is also some theoretical uncertainty on the far-IR/submm modeling of these sources.
Calibration Sources: Other Evolved Stars

Other stellar sources have also been used for years as mm/submm primary calibration sources. These include one particularly interesting source: MWC 349. This is a star with a nearly constant power law density falloff, resulting in a nearly constant spectral index from the IR to radio wavelengths, which goes like $\lambda^{-0.6}$. Its size is reasonable (~0.3"), and it is relatively non-variable. Furthermore, Jack Welch has measured this source absolutely at 30 GHz, and plans to measure it similarly at 90 GHz. It may therefore be a very good primary or secondary flux density calibrator.
Many solar system sources have enough flux density. There is a size problem, however:
We can correct, to some degree, for resolution effects, since we have a good idea of what the expected visibility function is. However, this only works to a certain degree. Must have enough short baselines to make the fitting accurate enough.
The zero-spacing flux density is the integral over the object. Assume circular projection (for illustration):

$$V_0^p = \left( \frac{2k}{\lambda^2} \frac{1}{D^2} \int_0^{2\pi} \int_0^R A(r, \psi) T_B^p(r, \psi) \, r \, dr \, d\psi \right) - \xi T_{CMB}$$

The brightness temperature as a function of location on the disk can be calculated, given a precise enough model of the atmosphere (if present) and surface (if probed).
Solid Surfaces: Theory

\[ T_B^p = \left(1 - R^p\right) \int_0^\infty k(x) \sec \theta_i T(x) e^{-\int_0^x k(x) \sec \theta_i x} \, dx \]

where \( R^p \) is the Fresnel reflectivity of the surface (as function of polarization \( p \)), \( k(x) \) is the absorption coefficient of the material in the subsurface as a function of depth, \( \theta_i \) is the incidence angle (angle between line of sight and surface normal), and \( T(x) \) is the physical temperature as a function of depth.
Solid Surfaces: Theory

The subsurface temperatures as a function of depth are found by solving the 1-D thermal diffusion equation:

\[
\frac{\partial}{\partial x} \left( K(x,T) \frac{\partial T}{\partial x} \right) = \rho(x)C(x,T) \frac{\partial T}{\partial t}
\]

where \( K(x,T) \) is the thermal conductivity as a function of depth and temperature, and \( \rho(x) \) and \( C(x,T) \) are the density and heat capacity of the material in the subsurface.
Solid Surfaces: Theory

In order to solve the diffusion equation, two boundary conditions are needed:

\[
\frac{\partial T}{\partial x} \bigg|_d = -\frac{J_0}{K_d}
\]

\[
\left(\frac{L_0}{4\pi D^2}\right)(1 - A_b)\sin^\psi - J_0 = \varepsilon_{LR}\sigma_B T_s^4 - K\frac{\partial T}{\partial x} \bigg|_s
\]
Solid Surfaces: Theory

So, the list of necessary parameters for the model are:

- $R^p$ - the surface Fresnel reflectivity
- $k(x)$ - the subsurface absorption coefficient
- $K(x,T)$ - the subsurface thermal conductivity
- $\rho(x)$ - the subsurface density
- $C(x,T)$ - the subsurface specific heat
- $J_0$ - the heat flow at depth
- $L_o$ - the solar luminosity
- $A_B$ - the surface Bond albedo (visible)
- $\varepsilon_{IR}$ - the surface IR emissivity
Atmospheres: Theory

\[ T_B = \int_0^\infty k(z)B(T(z))e^{\int_z^\infty \frac{k(z')}{\mu} \frac{dz'}{\mu}} \]

where \( k(z) \) is the absorption coefficient of the gases (or condensed phases) in the atmosphere, \( \mu \) is the cosine of the incidence angle, and \( T(z) \) is the physical temperature as a function of altitude.
Atmospheres: Theory

\[ k(z) = \sum \kappa_i(z) \]

so it is necessary to know what is in the atmosphere (abundances of the constituents), and the opacity of each constituent (the absorption coefficient can be written:

\[ k(z) = \rho(z) \kappa(z) \]

for density \( \rho \) and mass absorption coefficient \( \kappa \).
Atmospheres: Theory

When solving the microwave radiative transfer problem for planetary atmospheres, it is always (as far as I know) assumed that the temperature structure $T(z)$ is known – it is not solved for explicitly. It is generally taken from spacecraft occultations, or other sources, and assumed not to vary as a function of time. It is also generally not assumed to vary with location on the planet. How good are these assumptions? It is clear they are violated in some cases.
Problems: Polarization

For the solid surface bodies, the signal is intrinsically polarized as it passes through the surface-atmosphere interface. This can cause problems if it is not accounted for.

Mitchell 1993
BUT, this polarized signal can actually be used to determine the effective surface dielectric, which is needed for modeling the thermal emission. Form a polarized visibility of:

\[
V_p = \frac{\Re \{ V_{RL} + V_{LR} \} \cos 2\psi + \Im \{ V_{RL} - V_{LR} \} \sin 2\psi}{V_o}
\]

which is, theoretically:

\[
V_p(\beta) = \int_0^1 A(\rho) (R_\parallel - R_\perp) J_2(2\pi \rho \beta) \rho \, d\rho
\]

which can be inverted to find the dielectric.
Problems: Roughness

Surface roughness modifies both the total and polarized emission. For example, the polarized vis. fn. is modified:

It can be measured and modeled, but is another complication.
Problems: Atmospheric Lines

For the bodies with no (probed) surface, there is the possibility of “contamination” by atmospheric lines. If there are broad lines in the spectrum of the planet, and they are not properly modeled (which is difficult to do), then the flux density can be over- or under-estimated. In some cases, this effect can be as large as 10-15%. This is a problem for at least Saturn and Neptune.
Problems: Atmospheric Scattering

Furthermore, nearly all of these bodies have condensation (cloud or haze) layers, which will be probed at mm and submm wavelengths. Scattering within these layers provides an effective additional absorption source, plus a source for polarization (if multiple scattering is present). These layers are not well constrained and hence hard to model theoretically (e.g., size distribution of particles poorly known).
Mars

Mars is one of the best mm/submm primary flux density calibrators. The best current model is that of Don Rudy (current keepers are B. Butler and M. Gurwell). This model is good, but has several shortcomings:

- based fundamentally on cm scale (Baars et al.), since measurements were done at 2 & 6 cm at VLA;
- no roughness;
- uncertainties with surface CO$_2$ ice, extent & properties;
- no detailed surface albedo or emissivity information;
- no atmosphere.

In addition, Mars is a bit big (as large as 25").
The model takes into account the viewing geometry and martian season. Here are the models over one martian day and one martian year.
Uranus

Another very good calibrator, and smaller than Mars (most times, anyway), Uranus has been used as a primary calibrator for many years. The best models are those of Griffin and Orton, Moreno, and Hofstadter. It doesn’t suffer from extreme contamination from atmospheric lines, has little or no PH₃, etc… Gene Serabyn has some very accurate numbers for the brightness temperature (few %). There are, however, some problems with the model for it:

- $T(z)$ might be varying with time;
- cloud/haze layers?;
- constituent opacity uncertainties.
Weighting functions for frequencies of 1410, 675, 350, 90, 15, and 6 GHz. The methane cloud is probed at all frequencies, and the ammonia cloud is probed at 90 GHz.
The temperature structure of the deep atmosphere seems to be changing with time (Hofstadter & Butler 2002). Is this occurring higher up in the atmosphere?
Large Icy Bodies

The large icy bodies might be good choices for primary calibrators. These include: Galilean satellites, Titan, Triton, and smaller moons of Jupiter, Saturn, and Uranus.

Problems are:

- confusion from primary;
- less known physically about the surfaces/subsurfaces;
- mm emissivity problem for Ganymede (Muhleman & Berge 1991).

Titan gets around some of these problems and might be a very good choice.
Titan might be a very good primary calibrator, since it does not suffer from contamination from surface emission (in mm/submm, all emission effectively comes from atmosphere – exception is 35 GHz), and the uncertainties that come from it. There are still possible problems however:

- flux density not currently known to better than 10%;
- modeling effects of haze;
- atmospheric lines.
Asteroids are possible flux density calibrators. Larger asteroids (D > 150 km or so) are relatively spherical, and so have only weak light curves (few %), and can be well modelled (work by Lagerros and Müller). There are 34 such MBAs. They are relatively small, relatively strong (~ 100 mJy at 230 GHz, and going up like $\lambda^{-2}$), and have modellable light curves. They are not (in my opinion) good for primary flux density calibrators, but should be excellent secondary flux density calibration sources.
Conclusions

- Make decision on a priori vs. a posteriori (this might not happen until experience shows us how well we can do with a priori).
- Have to pick few true primaries, and probably need some more observations + theory. Good current candidates: MWC 349, Titan, Uranus, Mars.
- Decide on what to use for secondaries (probably QSOs and/or asteroids), and monitoring scheme for them.
- Will need good models of sky brightness distribution (I + pol’n) for all of them (primaries AND secondaries).
- 1% (or 3%, even) will still be extremely difficult.