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Imaging by an Optimizing Method

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Abstract. The imaging problem can be described as an optimizing problem in mathematics. Thus optimizing theory and algorithms can be used to solve it. In this paper we present an optimizing method, in which we take the imaging problem as an optimizing problem with linear constraints. We choose the objective function carefully. Both the mathematical expectation and the variance of the observed data are considered. Upper and lower limit source and background intensities can be conveniently considered. We adopt an algorithm very similar to the affine scaling approach in convex programming. Computer simulations of rotating modulation collimator imaging show that the quality of images from this method is better than that from the traditional cross-correlation method. Both point and extended sources can be imaged in the same field of view. We also apply the algorithm to ROSAT PSPC pointed observation data of the Crab nebula. The image quality is improved significantly. The extended structure of the Crab nebula can clearly be seen.

1. Introduction

The imaging problem may be described as inferring the sky brightness distribution from observations and prior knowledge (Cornwell 1992). In this paper, we will introduce an optimizing method and adopt it to the imaging problem. Then we will apply it to simulations of a rotating modulation collimator (RMC), and ROSAT PSPC (the Position Sensitive Proportional Counter) observations.

2. Imaging Problem

The imaging problem is:

$$d = Pf + n \tag{1}$$

where d is the observational data, P is the point spread function, f is the unknown sky, and n is the noise. Usually, there are some constraints for f and n in this linear system of equations. The optimizing problem is:

$$min.\ \ F(x) \tag{2}$$

$$subject\ to\quad Ax = b \tag{3}$$

$$x \geq 0 \tag{4}$$

where $F(x)$ is an objective function.

In astronomy, the noise n_k usually follows a Poisson or Gaussian distribution. Thus it has a certain expected value and variance. The sky intensity f may have a upper limit up and a lower limit low. Then we can make an objective function

$$F(f,n) = (\sum_i^k n_i^2/d_i - k)^2 + a(\sum_i^k n_i)^2 - b\sum_i^m(\ln(f_i - low_i) + \ln(up_i - f_i)) \quad (5)$$

where a and b are coefficients, k is the number of bins of observational data, and m is the number of sky bins. The constraint condition is

$$\sum_i^m p_{ji}f_i + n_j = d_j \quad (j = 1, ..., k) \quad (6)$$

Both f_i and n_j are unknown. This problem is similar to the convex programming problem.

3. Affine Scaling Algorithm

The affine scaling (AS) algorithm is one of the simplest and most efficient of interior point method algorithms (Dikin 1967). For the optimizing problem (2) $\sim$ (4) the AS algorithm in detail is:

1. Try to find an initial solution.
2. Calculate
$$H_k = [\nabla^2 f(x^k) + X_k^{-2}]^{-1} \quad (7)$$
$$[AH_kA^T]\omega^k = AH_k\nabla f(x^k) \quad (8)$$
$$s^k = \nabla f(x^k) - A^T\omega^k \quad (9)$$
3. Check whether the stopping criteria is satisfied.
4. Find a transition direction.
$$d_x^k = -H_k s^k \quad (10)$$
5. Calculate the step length α_k. Search for that α_k which minimizes the objective function.
6. Move to a new solution.
$$x^{k+1} \leftarrow x^k + \alpha_k d_x^k \quad (11)$$
7. Let $k \leftarrow k + 1$ and go to Step 2.

We developed an algorithm based on the affine scaling algorithm (Goldfarb 1991; Fang 1993) for problem (5) $\sim$ (6).

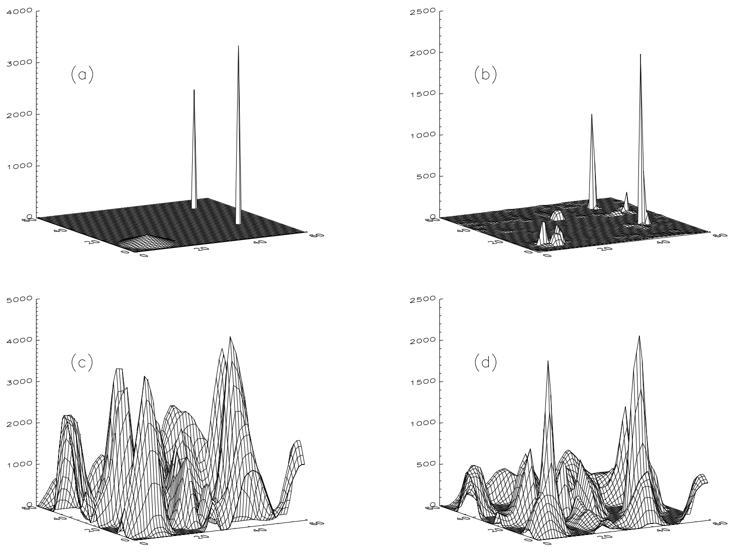

Figure 1. a) The assumed sources. b) Result of the AS algorithm. c) Result of cross-correlation. d) Result of CLEAN cross-correlation.

4. Application

4.1. Rotating Modulation Collimator

We have simulated a rotating modulation collimator. The configuration of the RMC is shown in Table 1. The background is assumed to be 0.09 ph cm^{-2} s^{-1}. The fluxes of the two point sources are assumed to be 1.0×10^{-2} and 6.7×10^{-3} ph cm^{-2} s^{-1}, and the total flux of the extended source 3.5×10^{-2} ph cm^{-2} s^{-1} (Figure 1a). The observing time is assumed to be one day. Figure 1b shows the

Table 1. The Configuration of the RMC

distance between strips (cm)	distance between grid planes (cm)	FOV (^{o})	total active area (cm^2)
1.0	34	6×6	1000

result of the AS algorithm, while Figure 1c and d result from the cross-correlation method and the CLEAN cross-correlation, respectively. Both the point sources and the extended source can be seen in Figure 1b. The angular resolution and image quality in Figure 1b are much better than in the cross-correlation images.

4.2. ROSAT PSPC Image of Crab

We used this algorithm to reconstruct ROSAT PSPC data. The results are shown in Figure 2. Figure 2a is the original image observed by PSPC. The

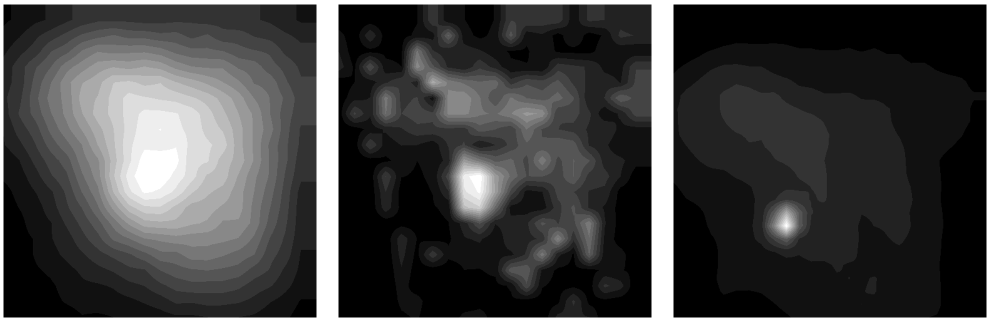

Figure 2. a) The original ROSAT PSPC Crab nebula image. b) The image obtained from the AS algorithm. The FOV is 100×100 arcsec. c) A ROSAT HRI image of the Crab nebula. The FOV is 100×96 arcsec.

result of the AS algorithm is shown in Figure 2b, where the extended structure is clearly seen. This extended structure can also be seen by ROSAT HRI (the High Resolution Imager) (Figure 2c).

5. Discussions and Conclusions

The calculation time for the AS algorithm depends on the initial solution. We can use the solution, from some other algorithm such as Richardson-Lucy iteration or cross-correlation, as the initial solution in order to reduce the calculation time.

In this paper we developed an AS algorithm and applied it to the imaging problem. The results show that this algorithm can be used in the imaging problem and usually results in a better image than that obtained from a traditional method such as cross-correlation. This algorithm can be also used in the data reconstruction of an imaging instrument (e.g., ROSAT PSPC).

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