

# Radiation from an Accelerated Charge

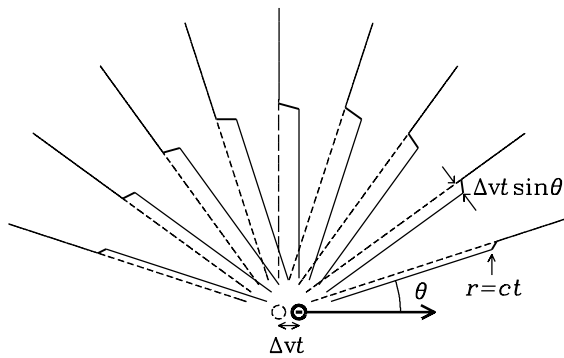
[see also Longair, High Energy Physics, 2nd edition, vol. 1, p. 64, and [Wikipedia](#)]

Maxwell's equations imply that all classical electromagnetic radiation is ultimately generated by accelerating electrical charges. It is possible to derive the intensity and angular distribution of the radiation from a point charge (a charged particle) subject to an arbitrary but small acceleration  $\Delta v/\Delta t$  via Maxwell's equations (and [retarded potentials](#)), but the complicated math obscures the physical interpretation that remains clear in J. J. Thomson's simpler derivation.

If a particle with electrical charge  $q$  is at rest, Coulomb's law implies that its electric field lines will be purely radial:  $\mathbf{E} = E_r$ . Suppose a charged particle is accelerated to some small velocity  $\Delta v \ll c$  in some short time  $\Delta t$ . At time  $t$  later, there will be a perpendicular component of electric field

$$\frac{E_{\perp}}{E_r} = \frac{\Delta v t \sin \theta}{c \Delta t},$$

where  $\theta$  is the angle between the acceleration vector and the line from the charge to the observer.



This figure shows the electric fields from an accelerated electron.

**Coulomb's law** for the radial component  $E_r$  of the **electric field** (electric force per unit charge) a distance  $r$  from a charge  $q$  is

$$E_r = \frac{q}{r^2} \quad (2D1)$$

in Gaussian cgs units. Substituting  $t = r/c$  gives

$$E_{\perp} = \frac{q}{r^2} \left( \frac{\Delta v}{\Delta t} \right) \frac{r \sin \theta}{c^2}$$

and

$$E_{\perp} = \frac{q\dot{v} \sin \theta}{rc^2} \quad (2D2)$$

This equation is valid for any small acceleration, not just a sinusoidal one. The delay time  $t$  has dropped out; the transverse field  $E_{\perp}$  instantaneously reflects the applied acceleration. Thus a sinusoidal acceleration would result in a sinusoidal variation with the same frequency in  $E_{\perp}$ . Note that  $E_{\perp} \propto r^{-1}$  in contrast to  $E_r \propto r^{-2}$ . Far from the charged particle (large  $r$ ), only  $E_{\perp}$  will contribute significantly to the radiation field. From the observer's point of view, only the visible acceleration perpendicular to the line of sight contributes to the radiated electric field; the invisible component of acceleration parallel to the line of sight does not radiate—what you see is what you get.

How much power is radiated in each direction? In a vacuum, the [Poynting flux](#), or power per unit area ( $\text{erg s}^{-1} \text{cm}^{-2}$ ) is

$$\vec{S} = \frac{c}{4\pi} \vec{E} \times \vec{H}$$

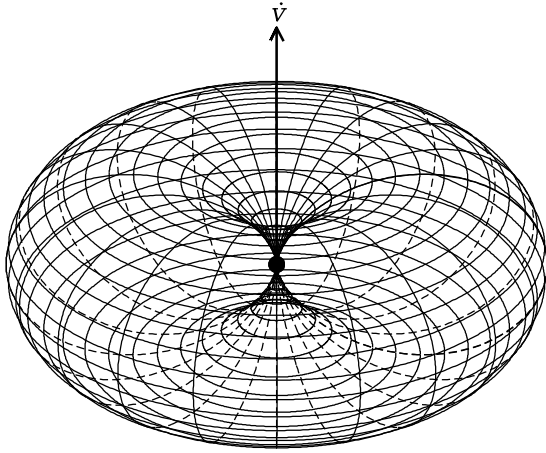
In cgs units  $|\vec{E}| = |\vec{H}|$  so

$$|\vec{S}| = \frac{c}{4\pi} E^2 \quad (2D3)$$

$$|\vec{S}| = \frac{c}{4\pi} \left( \frac{q\dot{v} \sin \theta}{rc^2} \right)^2$$

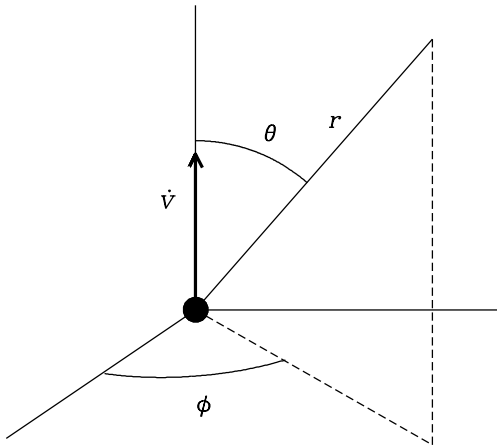
$$|\vec{S}| = \frac{1}{4\pi} \frac{q^2 \dot{v}^2 \sin^2 \theta}{c^3 r^2}$$

The charge radiates with a dipolar power pattern that looks like a doughnut whose axis is parallel to  $\dot{v}$ .



*The power pattern of Larmor radiation from a charged particle shown for an acceleration vector  $\dot{v}$  tilted  $60^\circ$  from the line of sight. The power received in any direction is proportional to the component of  $\dot{v}$  perpendicular to the line of sight.*

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*Coordinate system used to calculate the total power emitted by an accelerated charged particle.*

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The total power emitted is obtained by integrating over all directions:

$$P = \int_{\text{sphere}} |\vec{S}| dA$$

$$P = \frac{q^2 \dot{v}^2}{4\pi c^3} \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \frac{\sin^2 \theta}{r^2} r \sin \theta d\theta r d\phi$$

$$P = \frac{q^2 \dot{v}^2}{2c^3} \int_{\theta=0}^{\pi} \sin^3 \theta d\theta$$

[Evaluating this integral](#) gives  $\int_{\theta=0}^{\pi} \sin^3 \theta d\theta = 4/3$  so the total power emitted is

$$P = \frac{2}{3} \frac{q^2 \dot{v}^2}{c^3} \quad (2D4)$$

This result is called **Larmor's equation**. It states that any charged particle radiates when accelerated and that the total radiated power is proportional to the square of the acceleration. Since the greatest astrophysical accelerations are usually electromagnetic, the acceleration is usually proportional to the charge/mass ratio of the particle. Thus radiation from electrons is typically  $\approx 4 \times 10^6$  stronger than radiation from protons, which are  $\approx 2 \times 10^3$  times more massive.

Larmor's equation will be the basis for our derivations of radiation from a short dipole antenna as well as for free-free and synchrotron emission from astrophysical sources. Beware that Larmor's formula is nonrelativistic; it is valid only in frames moving at velocities  $v \ll c$  with respect to the radiating particle. To treat particles moving at nearly the speed of light in the observer's frame, we must use Larmor's equation to calculate the radiation in the particle's rest frame and then transform the result to the observer's frame in a relativistically correct way. Also, Larmor's formula does not incorporate the constraints of quantum mechanics, so it should be applied with great caution to microscopic systems such as atoms. For example, Larmor's equation incorrectly predicts that the electron in a hydrogen atom will quickly radiate away all of its kinetic energy and fall into the nucleus.