Formation of Rotationally Supported Protostellar Disks: Some Theoretical Difficulties
Protostars and disks

Inside a protostellar core, a star is forming – surrounded by a protostellar accretion disk, and the dense parts of the core.

Outflows – winds and jets – are also produced.
Hydrodynamic model of disk formation

Gas motions in the core have angular momentum. Its conservation allows disk formation.

Axisymmetric hydrodynamic simulation at $t=10^{12}$ s~$3\times10^4$ yr

Prominent 400 AU disk of 0.1 Msun around a 0.5 Msun protostar. Rotationally supported – Keplerian. Subsonic, very dense. Surrounded by a rapidly accreting supersonic flattened structure.

Angular momentum transport is a problem in this model
Magnetized Models

- Necessary: Dynamically significant \( B \) fields are observed in molecular cloud cores.
  
  Example: SMA observation of polarized dust emission →

- Magnetism solves some problems
  
  • It provides a mechanism for outflows – winds and jets (Blandford & Payne 1982). Simulations show this magneto-centrifugal mechanism works (e.g. Ustyugova et al 1999, Krasnopolsky et al 1999)
  
  • Magnetism can provide the torques needed for angular momentum transport – magnetic braking – allowing accretion of mass to the central object (e.g., Basu & Mouschovias 1995, Krasnopolsky & Königl 2002)

- However, magnetic braking can become excessive – leaving too little angular momentum for a disk to form (Mellon & Li 2008, 2009). “Magnetic Braking Catastrophe”
Example of Excessive Magnetic Braking: 2D Axisymmetric Ideal MHD

Magnetic braking acts the strongest in a model without explicit diffusion (Ideal MHD)

Result:
Powerful supersonic accretion takes place in blobs and rings.
Not equatorially symmetric, not rotationally supported.
Dominated by magnetic reconnection events – numerically mediated.

NO KEPLERIAN DISK
Mechanism of Magnetic Braking

### Introduction

- Slower rotation
- Faster rotation

- Collapse pinches the poloidal field $B_p$ into a tight hourglass
- Differential twist of field lines generates a toroidal field $B_\varphi$
- Magnetic tension of twisted field yields a braking torque on the faster-rotating inner material
- Braking rate $\propto B_p \times B_\varphi$
- The stronger the field, the harder to form disks
Need to weaken magnetic braking

- Difficulty with disk formation in ideal MHD: magnetic flux freezing → magnetic split monopole → excessive braking

- Reducing the $B$ field? It will not help by much: the simulation used a field that is pretty typical ($B_0=35\mu$G, dimensionless mass-to-flux ratio $\lambda \sim 3$).

- We can try to weaken the coupling of the magnetic field to matter, utilizing non-ideal MHD effects. These effects allow matter to fall in without having to drag all of the magnetic field with it. As a bonus, the non-ideal MHD effects also avoid the so-called “Magnetic flux problem”.

- We will consider three non-ideal MHD effects: Ohmic resistivity, the Hall effect, and ambipolar diffusion.

- Then we will consider if 3D effects can save the disk.

Introduction
Classical resistivity

An inner, denser flattened structure forms.
Fragmented, and far from being rotationally supported.
Accretion is mostly supersonic.
Magnetic tension allows for some subsonic accretion rings.
Inner structure still dominated by not well-resolved reconnection events.

NO KEPLERIAN DISK

Need to try with enhanced resistivity
Enhanced resistivity enables disk formation

$t=10^{12} \text{ s}$

DISK

$B_0=35 \mu \text{G}$

Enhanced resistivity, $\eta=10^{20}\text{cm}^2/\text{s}$

Result: Very dense Keplerian disk, growing with time. Surrounded by a pseudodisk supported by magnetic tension.

How much resistivity $\eta$ do we need?
Exploring enhanced $\eta$ and $B$

$\eta=3\times10^{19}\text{cm}^2/\text{s}$
$B_0=35\mu\text{G}$: Tiny

$\eta=10^{19}\text{cm}^2/\text{s}$
NO DISK

$\eta=3\times10^{17}\text{cm}^2/\text{s}$
Magnetic Blob

$\eta=10^{17}\text{cm}^2/\text{s}$
NO DISK

$\eta=3\times10^{19}\text{cm}^2/\text{s}$
$B_0=10\mu\text{G}$

$\eta=10^{19}\text{cm}^2/\text{s}$
$B_0=10\mu\text{G}$
DISK

$\eta=10^{18}\text{cm}^2/\text{s}$
$B_0=10\mu\text{G}$
TINY

$\eta=3\times10^{17}\text{cm}^2/\text{s}$
$B_0=35\mu\text{G}$: Blob Movie

Enhanced Ohmic Resistivity
Ohmic Resistivity: Summary

• Classical resistivity is unable to weaken magnetic braking enough to allow a rotationally supported disk (for a realistic magnetization).

• Enhanced resistivity allows disk formation

  • Need about $\eta=3 \times 10^{19} \text{cm}^2/\text{s}$ to form a disk larger than 10AU for $\lambda \sim 3$, and about $\eta=10^{18} \text{cm}^2/\text{s}$ for $\lambda \sim 10$.

  • Need to explore mechanisms that produce enhanced resistivity. Turbulent resistivity (e.g. Lubow et al. 1994, Guan & Gammie 2009). Current-driven instabilities (e.g. Norman&Heyvaerts 1985). Reconnection diffusion in turbulent flows (e.g. Lazarian 2012, Santos-Lima et al. 2010 and 2012).

Can non-ideal MHD effects save the disk in 2D? (Li, Krasnopolsky & Shang 2011)

Large grain size: 1 μm

AD dominates over most densities

Power-law size distribution w/ small grains (MRN) (Mathis, Rumpl & Nordsieck 77)

Smaller grain, Hall more important
Ambipolar diffusion & magnetic flux redistribution

(Li, Krasnopolsky & Shang 2011)

- 2D collapse of initially uniform, $\lambda=2.9$ non-rotation core, with only AD (including MRN grains), spherical coord. using ZEUS–TW
- Split monopole is avoided
- Magnetic flux piles up outside star: high magnetic tension, slow down collapse (Li & McKee 96, see also Ciolek & Königl 98, Krasnopolsky & Königl 02, Tassis & Mouschovias 07)
Can ambipolar diffusion save the rotationally supported disk?

- No! No rotationally supported disk (see also Krasnopolsky & Königl 02)

Almost complete braking of rotation

Magnetically induced disk!
Can ambipolar diffusion, Ohmic dissipation & Hall effect save the rotationally supported disk? Probably not

- Hall spin-up
- Bending of poloidal B → jφ
- In simple e-ion fluid, e carries current → vφ
- e tied to B → Bφ
  - Magnetic torque in φ-dir
- Flip B, flip jφ, vφ, Bφ, torque

Non-ideal effects unable to save disks
We carried out 3D simulations of collapse including three non-ideal processes: AD, enhanced Ohmic dissipation, and decoupling at the inner boundary at $r=10^{14}$ cm.

Result: the inner protostellar accretion flow is driven unstable by the magnetic flux decoupled from the matter that enters the central object. When this interchange instability is fully developed, the flow structure becomes highly filamentary, as a result of the interplay between gravity-driven infall and magnetically-driven expansion. In particular, the AD shocks found in 2D are unstable.

3D collapse simulation with AD ($\zeta=9\times10^{-17}/s$), at a time when $M=0.092M_{\odot}$. Left panels: equatorial plane (unit $\mathbf{v}$ vectors in white); right panels: a meridian plane (with unit $\mathbf{B}$ vectors). Top panels: log($\rho$); bottom panels: log plasma $\beta$, with $\beta=1$ in white.
Growth of the 3D instability is clearly seen in these models including a step-function resistivity ($\eta$ goes from 1 to $10^{19}$ cm$^2$/s for $r<2 \times 10^{14}$ cm). Models I and J incorporate also AD. Model J has initial rotation; that does not change the outcome of the instability, and no RSDs are seen.

3D Instability
3D Instability: Summary

• Magnetic interchange instabilities are seen to take place during collapse once the axisymmetry assumption is released.

• Magnetic flux is transported by macroscopic advection, in addition to microscopic diffusion.

• Diffusive effects are important to this process, in that they provide the initial decoupling needed for the instability to start; after decoupling, more strongly magnetized regions expand away along some azimuthal directions, while less magnetized regions sink in.

• The instabilities lower B close to the protostar; however, magnetic braking is still efficient, and no RSDs were observed in this set of simulations.

Can magnetic field-rotation axis misalignment enable large-scale disk formation?

- Answer from simulations of Joos, Hennebelle & Ciardi (2012): it depends (also Price+Bate07, Hennebelle+Ciardi09)

- Answer from Li, Krasnopolsky & Shang (2013): Yes & No

  - Yes for weak-field cases of $\lambda \sim 14$ and $7$, where disks form in the orthogonal but not aligned case

  - No for moderately strong-field case of $\lambda \sim 4$, where there is NO disk even in the orthogonal case!
Weak field (λ_{eff}∼14), aligned case where disk is suppressed.
Weak field ($\lambda_{\text{eff}} \sim 14$), orthogonal case where a disk forms.
Why does misalignment help with disk formation in weakly magnetized cores? A key difference: outflow

Aligned case with $\theta=0^0$

Orthogonal case with $\theta=90^0$
Why does misalignment help with disk formation in weakly magnetized cores? – Key: outflow

Aligned case

Orthogonal case: suppression of outflow → less a.m. removal → disk

Magnetic field lines

Isodensity surface
Moderate field ($\lambda_{\text{eff}} \sim 4$), disk suppressed even in orthogonal case.
Summary: a fraction of cores may have large-scale disks enabled by misalignment (Krumholz+13; Li+13)

- Median $\lambda \sim 2$ for dense cores inferred by Troland+Crutcher08
- Half cores with $\lambda > 2$, capable of disk formation for large misalignment according to Joos+12
- If misalignment random (Hull+12), half cores with $\theta > 60^0$
  \[ \Rightarrow \text{disk fraction} \sim \frac{1}{2} \times \frac{1}{2} \sim \frac{1}{4} \]
- However, if only cores with $\lambda > 4$ form disks (Li+13)
  \[ \Rightarrow \text{disk fraction reduced by a factor of} \sim 2 \]
  \[ \Rightarrow \text{disk fraction} \sim 10\% \]
- Majority of cores do not produce large-scale disks through this mechanism?
Summary: difficulty with disk formation


- **Microscopic non-ideal MHD may be not strong enough in 2D.** Machida+(2007) and Dapp+(2012) showed that Ohmic dissipation can enable small (AU scale) disks. Enhanced resistivity (KLS2010) and strong Hall effect effect (KLS2011) allow 100AU scale disks, but the microscopic values of $\eta$ and $Q$ do not seem large enough (LKS2011) in the larger scale, while AD acts to increase magnetic braking (Mellon & Li 2009, Krasnopolsky & Königl 2002).

- Protostellar accretion flows can be unstable to magnetic interchange instability driven by flux redistribution (KLS2012). Trapped fieldlines make disk formation difficult.

→ **Disk formation not as trivial as often believed** (basic problem: magnetic flux concentration by accretion)
Summary: possible resolutions

- Weak core magnetization probably not consistent with available data

- **Misalignment weakens magnetic braking.** (Hennebelle & Ciardi 2009). Might be not enough to enable large disk formation in majority of dense cores.

- Turbulence may facilitate disk formation in various plausible conditions, such as favorable patterns of turbulent flow (Seifried+2012), and turbulent enhancement of magnetic reconnection (Santos-Lima, Gouveia dal Pino, & Lazarian 2012)

- Outflow may weaken magnetic braking by stripping away the slowly-rotating envelope (Mellon&Li 2008, Machida+2012). Not quantified yet.

→ **Problem of disk formation remains unsolved**
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<th>Advantages</th>
<th>Disadvantages</th>
<th>Observation</th>
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<tr>
<td>B/Ω misalignment</td>
<td>Numerically proven</td>
<td>Needs $\lambda &gt; [a \text{ few}]$, and misalignment</td>
<td>B/envelope rotation misalignment</td>
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<tr>
<td>Ohmic decoupling</td>
<td>Works (at large $\eta$)</td>
<td>May need high $\rho$ such as found at $&lt;1\text{AU}$</td>
<td>Look for small RS disks in Class 0: <strong>ALMA</strong></td>
</tr>
<tr>
<td>Envelope depletion through either ↑outflows or ↓infall</td>
<td>Expected to allow disks to grow</td>
<td>Still unproven. May require a small seed disk.</td>
<td>RS disk growth about the Class 0/1 transition.</td>
</tr>
<tr>
<td>Hall torque</td>
<td>Works (at large $Q_H$)</td>
<td>Requires very large $Q_H$ (dust grains may help).</td>
<td>About 50% envelope/disk counter-rotation: <strong>ALMA</strong></td>
</tr>
<tr>
<td>Turbulent reconnection diffusion</td>
<td>Basic limitations of ideal MHD. Simulations work.</td>
<td>Artificial simulations: numerical diffusion, turbulence driving</td>
<td>Specific kind of turbulence Lab experiments may help with theory problems.</td>
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<tr>
<td>Gravitational torques</td>
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<tr>
<td>MHD instabilities</td>
<td>Universal</td>
<td>seems counterproductive</td>
<td>?</td>
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<tr>
<td>Ambipolar diffusion</td>
<td>Solves B flux problem</td>
<td>seems to increase braking</td>
<td>?</td>
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<td>Weak field</td>
<td>HD and nearly HD models work</td>
<td>Needs very large $\lambda$</td>
<td>Very small field intensities</td>
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<tr>
<td>Your own imagination</td>
<td>Unlimited possibilities</td>
<td>Not always works</td>
<td>Submit the proposal !</td>
</tr>
<tr>
<td>Nature itself</td>
<td>Makes disks reliably</td>
<td>We do not know for sure how that’s done</td>
<td>SEARCH FOR EARLY CLASS 0 DISKS WITH ALMA</td>
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