Interferometry Wideband, Widefield Calibrator Models

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Abstract—Much of the calibration of radio interferometers depends on observations of celestial objects of known properties. Most such celestial objects have a flux density that varies with frequency; for continuum sources this is typically close to a power law. Wideband interferometers such as the EVLA thus require wideband models of calibrators for accurate calibration of visibility data. Such models need to represent the structure of the calibrator as a function of frequency. In addition, at low frequencies other sources may also be in the array beam and need to be included in the model. The description of a wideband, widefield calibrator model is given in this memo in the form of a FITS image containing an “AIPS CC” table. Example FITS headers are given and a repository of EVLA calibration images maintained.

Index Terms—interferometry, calibration

I. INTRODUCTION

Calibration of radio interferometers frequently involves observations of celestial objects of known properties; these are referred to as “calibrators”. Good photometric calibrators tend to be physically large and are frequently resolved by interferometers. Adequate description of such calibrators includes the structure of the object. These calibrators have structures that vary with frequency; typically power laws with flux densities decreasing with increasing frequency. Thus, the description of such sources includes the variation of structure with frequency.

At low frequencies, ∼GHz and below, there is an additional complication that the receptor pattern of the interferometer elements is sufficiently broad that other sources in the field of the calibrator contribute a non trivial fraction of the emission in the field. For proper calibration, these other sources need to be included in the model of the calibrator. In addition, these additional sources may be sufficiently removed from the calibrator that a two dimensional representation of the relative position is inadequate.

This memo describes a working implementation of such a wide-field, wide-band calibrator model expressed as a FITS image containing a modified “AIPS CC” sky model table. The implementation described here is in the Obit package [1] 1

II. SOURCE STRUCTURE AND VISIBILITIES

Celestial source structures are generally expressed in terms of a linear combination of some basis function. The commonly used CLEAN technique uses a set of delta functions and/or Gaussians to model the source structure. The relationship between the sky model and observable visibilities (Vj) as a function of frequency (ν) is:

\[ V_{j,\nu} = \sum_{k=0}^{n} A_{j,k,\nu} e^{-2\pi i \nu (\frac{u_{j0} x_k + v_{j0} y_k + w_{j0} z_k)}{(u_{j0}^2 + v_{j0}^2 + w_{j0}^2)}} \]  

where \( u_{j0}, v_{j0} \) and \( w_{j0} \) are the baseline coordinates in wavelengths at the reference frequency, \( \nu_0 \), and the sky model consists of \( n \) components with visibility amplitude \( A_{j,k,\nu} \) at relative celestial position given by the direction cosines \( x_k, y_k \) and \( z_k \). The function \( A_{j,k,\nu} \) depends on the basis function and for delta functions is

\[ A_{j,k,\nu} = s_{k,\nu} \]

where \( s_{k,\nu} \) is the flux density of component \( k \) at frequency \( \nu \). For elliptical Gaussians, the visibility amplitude is

\[ A_{j,k,\nu} = s_{k,\nu} e^{\alpha u_{j0}^2 + \beta v_{j0}^2 + \gamma u_{j0} v_{j0}} \]

where

\[ a = -2\pi^2 ((\cos(\phi) \sigma_{maj})^2 + (\sin(\phi) \sigma_{min})^2) \]
\[ b = -2\pi^2 ((\sin(\phi) \sigma_{maj})^2 + (\cos(\phi) \sigma_{min})^2) \]
\[ c = -4\pi^2 \cos(\phi) \sin(\phi)(\sigma_{maj}^2 - \sigma_{min}^2) \]

\( \phi \) is the position angle of the major axis of the ellipse, \( \sigma_{maj} \) is the size (radians) of the standard deviation of the major axis of the ellipse and \( \sigma_{min} \) is the size of standard deviation of the minor axis of the ellipse. \( u_{j\nu} \) and \( v_{j\nu} \) are the baseline coordinates at frequency \( \nu \) which are given by \( \frac{x_k}{\nu_0} u_{j0} \) and \( \frac{y_k}{\nu_0} v_{j0} \).

For optically thin uniform spheres, the visibility amplitude is

\[ A_{j,k,\nu} = s_{k,\nu} \left( \frac{\sin(a) - \cos(a)}{a^2} \right) \]

where \( a = 3.04617 \times 10^{-5} (u_{j0}^2 + v_{j0}^2) r \) and \( r \) is the radius of the sphere in degrees.

III. SOURCE SPECTRUM

The technique presented here uses the following adaptation of the traditional representation of a continuum source spectrum:

\[ s_\nu = s_\nu_0 e^{\alpha \log(\nu/\nu_0)} + \beta \log(\nu/\nu_0)^2 + ... \]  

where \( s \) is the spectral flux density and \( \alpha, \beta \ldots \) are the “spectral index” and one or more curvature terms. As many terms can be used in the exponent as are needed to accurately represent the observed emission although for most practical cases of limited bandwidth the spectral index is sufficient.
IV. IMPLEMENTATION

Faceted imaging as is used in Obit and AIPS, produces a sky model divided into a number of flat facets on the sky to approximate the effects of the curvature of the sky. These individual facets are then “flattened”, re-projecting if necessary, onto a single image. The CLEAN sky model components can also be collected into a single table. Since the CLEAN sky model of a given facet is wrt that facet, the combination of CLEAN sky models must incorporate any facet dependent geometry into the component description prior to combination into a single table.

The procedure implemented in Obit (ObitTableUtil:ObitTableCCUtilAppendShift) uses a modified implementation of the AIPS CC table. The modifications are:

- Addition of a third “offset”, column “DELTAZ” in the z direction.
- Implementation of multiple basis functions and spectral descriptions in the “PARMS” array column.

The reference position in the image header is the celestial position from which the sky model component are offset.

The optional PARMS array column has been traditionally used in AIPS to describe components of basis functions. The first three elements are for component parameters. For elliptical Gaussians, these are the FWHM of the major axis, minor axis and position angle (all in degrees). For optically thin uniformly filled spheres, the first is the radius in degrees. The fourth element gives the basis function type and the usage in Obit adds the type of spectral information. The basis function types are:

- 0 = point.
- 1 = elliptical Gaussian.
- 2 = optically thin uniform sphere.

Subsequent elements in the PARMS array give the spectral information. There are two representations of the component spectrum; first, the “spectrum”, in which the component’s spectrum is parameterized in terms of the $\alpha$, $\beta$ ... described in Equation 2. Second, a “tabulated spectrum” in which the flux density is given in an array of frequency bins. Only the “spectrum” representation is used in the standard calibration flux density is given in an array of frequency bins. Only the tabulated.

For tables containing the “spectrum” representation, the number of parameters ($\alpha$, $\beta$ ...) is the length of the PARMS array minus 4 as well as the image header keyword “NTERM”. These tables may be recognized by a value between 10 and 19 in the 4th element of the PARMS array. The reference frequency for the spectrum is the reference frequency of the associated image.

Interpretation of the “tabulated spectrum” representation requires additional information from the file image descriptor. These are keywords in either the FITS or AIPS form and for Obit objects are in the myDesc->info member. These items are:

- The axis type on the frequency axis is “SPECLNMF”.
- Keyword “NSPEC” gives the number of frequency bins tabulated.
- Keyword “FREQ...n” for n in [1,NSPEC] zero padded to 8 characters is the central frequency (Hz) for the bin.
- Keyword “FREL...n” is the lower bound of frequency for the bin.
- Keyword “FREH...n” is the upper bound of frequency for the bin.

These tables have a value of the 4th element of the PARMS array of between 20 and 29.

V. COMBINED CALIBRATOR SKY MODEL TABLE

In the general case, multiple facets need to be combined into a single sky model table. The parameters for the model components are described in the following in terms of entries in an “AIPS CC” FITS table. The entry for the “FLUX” column for a given model component is the flux density at the reference frequency; primary beam corrections should NOT be applied. For Gaussian basis functions, the first three entries in the “PARMS” array for each table row are 1) FWHM of the major axis (deg), 2) the FWHM of the minor axis and the orientation on the sky of the major axis (deg). The conversion factor from a Gaussian $\sigma$ to FWHM is 2.35482. For point components, these values are all 0. The 4th element of the “PARMS” array is 10 for point components, 11 for elliptical Gaussians and 12 for optically thin uniform spheres. Subsequent elements in the “PARMS” array are the $\alpha$, $\beta$ ... spectral terms.

The sky model component position parameters in Eq 1, $x_k$, $y_k$ and $z_k$, are offsets with respect to a reference celestial position. These sky model components are located on the celestial sphere but are derived from a flat image tangent to this sphere so in general, require three components to describe. $x$ is east, $y$ is north and $z$ is towards the earth; these are the values represented by the “DELTAX”, “DELTAY” and “DELTAZ” columns in the combined calibrator sky model table. The derivation of the “DELTA?” column entries in the combined table depends on the projection type of the image. When combining the components from multiple facets, the offsets $\Delta x_k$, $\Delta y_k$ (“DELTAX” and “DELTAY” columns in the facet model table) are first adjusted to the facet center:

$$ x_{pk} = \Delta x_k - x_{coff} $$
$$ y_{pk} = \Delta y_k - y_{coff} $$

where $k$ indicates the component and $x_{coff}$ and $y_{coff}$ are the correction (RA, dec.) in degrees between the reference position of the facet from which component $k$ was obtained and its center. These offsets are the Euclidian distance (number of pixels times the cell spacing) between the reference position and the facet center. These are only nonzero for faceted imaging using a common tangent point. Then, a rotation is applied for the projection used to make the facet

$$ xyz_{k,0} = x_{pk} \text{umat}_{0,0} + y_{pk} \text{umat}_{1,0} $$
$$ xyz_{k,1} = x_{pk} \text{umat}_{0,1} + y_{pk} \text{umat}_{1,1} $$
$$ xyz_{k,2} = x_{pk} \text{umat}_{0,2} + y_{pk} \text{umat}_{1,2} $$
where $\text{umat}$ is a rotation matrix used to project data from the combined reference position to the reference position of the facet of component $k$. This matrix depends on the details of the image projection and is discussed below.

The offsets between the facet center and combined reference positions are derived from the directions cosines.

\[
x_{\text{off}} = r(\cos(r_i) + m\sin(r_i)),
\]
\[
y_{\text{off}} = r[-l\sin(r_i) + m\cos(r_i)],
\]
\[
z_{\text{off}} = r[\sqrt{1 - l^2 - m^2} - 1]
\]

where the factor $r$ converts radians to degrees.

**B. SIN projection with common tangent plane faceting**

The other SIN faceting projection discussed here is one in which the facets are re-projected onto the common tangent plane as described by [2]. In Obit, this is referred to as “2D” imaging. The $\text{umat}$ rotation matrix for this projection is given in the following.

The projection matrix between the combined reference position and that of the facet is

\[
\begin{bmatrix}
1 & 0 & -l/d \\
0 & 1 & -m/d \\
0 & 0 & -1
\end{bmatrix}
\]

where $l = \cos(\delta)\sin(\alpha - \alpha_r)$, $m = \sin(\delta)\cos(\delta_r - \cos(\delta)\sin(\delta_r)\cos(\alpha - \alpha_r)$, and $d = \sqrt{1 - l^2 - m^2}$.

The $\text{umat}$ matrix is the product of these

\[
\text{umat} = \text{ri} \text{ rp} \text{ rc}.
\]

C. NCP projection

Images made with strict east-west arrays can benefit from using NCP projection where the image is projected onto a plane tangent to the celestial sphere at the north celestial pole. Faceting may or may not be used for NCP projection as it is not needed for the sky curvature. The $\text{umat}$ rotation matrix for this projection is the unity matrix

\[
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

The offsets between the facet center and visibility reference positions are derived from the direction cosines.

\[
l = \cos(\delta)\sin(\alpha - \alpha_r),
\]
\[
m = \cos(\delta)\sin(\delta_r)\cos(\alpha - \alpha_r)
\]
VI. INTERPRETATION OF MODEL TABLE

The contents of the AIPS CC table can be used with Eq. 1 to calculate visibilities corresponding to the model. $s_{k,0}$ is the “FLUX” column value of the kth row; the first three elements of the “PARMS” array give the size and orientation of the Gaussian and/or optically thin spheres. The FWHM values in the table are converted to Gaussian $\sigma$s by the factor 1/2.35482 and then converted from degrees to radians. The 4th element of the PARMS array gives the basis and spectral type and the subsequent columns give the $\alpha$, $\beta$... values. The $x_k$, $y_k$, and $z_k$ parameters are derived from the “DELTAX” ($\Delta x$), “DELTAY” ($\Delta y$) and “DELTAZ” ($\Delta z$) columns of row k as:

$$x_k = d2r \Delta x + x_{off},$$
$$y_k = d2r \Delta y + y_{off},$$
$$z_k = d2r \Delta z + z_{off}$$

where $d2r$ is the conversion factor from degrees to radians and $x_{off}$, $y_{off}$, and $z_{off}$ account for any offset between the reference positions of the calibrator model and visibility data for which the model values are to be calculated. The details depend on the projection type of the visibility data.

A. SIN projection

The direction cosines of the relative positions of the calibrator model and visibility data-sets are given by

$$l = \cos(\delta_r) \sin(\alpha_r - \alpha_v),$$
$$m = \sin(\delta_r) \cos(\delta_v) - \cos(\delta_r) \sin(\alpha_r - \alpha_v) \cos(\alpha_r - \alpha_v).$$

where $\alpha_v$ and $\delta_v$ are the reference Right ascension and declination of the visibility data. The offset parameters are then

$$x_{off} = +l \cos(r_i) + m \sin(r_i),$$
$$y_{off} = -l \sin(r_i) + m \cos(r_i),$$
$$z_{off} = \sqrt{1 - l^2 - m^2} - 1$$

where $r_i$ is the relative rotation angle between the calibrator model and visibility data on the sky.

B. NCP projection

The direction cosines of the relative positions of the calibrator model and visibility data-sets are given by

$$l = \cos(\delta_r) \sin(\alpha_r - \alpha_v),$$
$$m = \{\cos(\delta_v) - \cos(\delta_r) \cos(\alpha_r - \alpha_v)\} / \sin(\delta_v)$$

and offsets

$$x_{off} = +l \cos(r_i) + m \sin(r_i),$$
$$y_{off} = -l \sin(r_i) + m \cos(r_i),$$
$$z_{off} = 0$$

VII. EXAMPLE

The FITS image and AIPS CC table headers for an example model file are given in the appendix and a few example rows are given in Table I. This example contains only point components and only the flux density and spectral index are given. A set of calibrator images for the EVLA is maintained in ftp://ftp.cv.nrao.edu/NRAO-staff/bcotton/Obit/CalModels.
Example FITS image header for file containing modified AIPS CC table.

```
SIMPLE = T / file does conform to FITS standard
BITPIX = 32 / number of bits per data pixel
NAXIS = 4 / number of data axes
NAXIS1 = 201 / length of data axis 1
NAXIS2 = 201 / length of data axis 2
NAXIS3 = 1 / length of data axis 3
NAXIS4 = 1 / length of data axis 4
EXTEND = T / FITS dataset may contain extensions

COMMENT FITS (Flexible Image Transport System) format is defined in 'Astronomy and Astrophysics', volume 376, page 359; bibcode: 2001A&A...376..359H

CTYPE1 = 'RA---SIN' / Axis type
CDELT1 = -2.543136E-04 / Axis coordinate increment
CRPIX1 = 1.010000E+02 / Axis coordinate reference pixel
CROTA1 = 0.000000E+00 / Axis coordinate rotation
CRVAL1 = 2.027845332653E+02 / Axis coordinate value at CRPIX

CTYPE2 = 'DEC--SIN' / Axis type
CDELT2 = 2.543136E-04 / Axis coordinate increment
CRPIX2 = 1.010000E+02 / Axis coordinate reference pixel
CROTA2 = 0.000000E+00 / Axis coordinate rotation
CRVAL2 = 3.050915523770E+01 / Axis coordinate value at CRPIX

CTYPE3 = 'FREQ ' / Axis type
CDELT3 = 1.658036E+07 / Axis coordinate increment
CRPIX3 = 1.000000E+00 / Axis coordinate reference pixel
CROTA3 = 0.000000E+00 / Axis coordinate rotation
CRVAL3 = 3.430400000000E+08 / Axis coordinate value at CRPIX

CTYPE4 = 'STOKES ' / Axis type
CDELT4 = 1.000000E+00 / Axis coordinate increment
CRPIX4 = 1.000000E+00 / Axis coordinate reference pixel
CROTA4 = 0.000000E+00 / Axis coordinate rotation
CRVAL4 = 1.000000000000E+00 / Axis coordinate value at CRPIX

OBSRA = 2.027845332653E+02 / Observed Right Ascension
OBSDEC = 3.050915523770E+01 / Observed declination

OBJECT = 'J1331+30' / Name of object
TELESCOPE = 'EVLA ' / Telescope used
INSTRUME = 'EVLA ' /
OBSERVER = 'Richard ' /
DATE-OBS = '2014-02-16' / Date (yyyy-mm-dd) of observation
DATE-MAP = '2014-03-31' / Date (yyyy-mm-dd) created
ORIGIN = 'Generated by Obit' /
EPOCH = 2.000000E+03 / Celestial coordinate epoch

BUNIT = 'JY/BEAM ' / Image pixel units
ALTRPIX = -5.000000E+00 / Alternate reference pixel
VELREF = 3 / >256 radio, 1 LSR, 2 Hel, 3 Obs
CLEANBMJ = 1.508745E-03 / Convolving Gaussian major axis FWHM (deg)
CLEANBMN = 1.301661E-03 / Convolving Gaussian minor axis FWHM (deg)
CLEANNPA = 8.384478E+01 / Convolving Gaussian position angle (deg)
CLEANNIT = 3824 / Number of Clean iterations

D03D = 1 /
XFXOFOFF = 0.00000000000000E+00 /
YFXOFOFF = 0.00000000000000E+00 /
QUANT = 6.38142200000000E-04 /
NTERM = 1 /
```
BLANK = -2147483648 / Blanking value
BScale = 0.0006381422 / Data scaling value
BZERO = 0. / Data offset value
HISTORY AIPS CLEAN BMAJ= 1.5087e-03 BMIN= 1.3017e-03 BPA= 83.84
HISTORY AIPS CLEAN NITER= 3824
HISTORY AIPS IMAGE ITYPE=1 XPOFF= 0.00000000 YPOFF= 0.0000000

Example FITS table header for modified AIPS CC table.

XTENSION= ’BINTABLE’ / binary table extension
BITPIX = 8 / 8-bit bytes
NAXIS = 2 / 2-dimensional binary table
NAXIS1 = 36 / width of table in bytes
NAXIS2 = 1430 / number of rows in table
PCOUNT = 0 / size of special data area
GCOUNT = 1 / one data group (required keyword)
TFIELDS = 5 / number of fields in each row
TTYPE1 = ’FLUX ’ / label for field 1
TFORM1 = ’1E ’ / data format of field: 4-byte REAL
TUNIT1 = ’JY ’ / physical unit of field
TTYPE2 = ’DELTAX ’ / label for field 2
TFORM2 = ’1E ’ / data format of field: 4-byte REAL
TUNIT2 = ’DEGREE ’ / physical unit of field
TTYPE3 = ’DELTAY ’ / label for field 3
TFORM3 = ’1E ’ / data format of field: 4-byte REAL
TUNIT3 = ’DEGREE ’ / physical unit of field
TTYPE4 = ’DELTAZ ’ / label for field 4
TFORM4 = ’1E ’ / data format of field: 4-byte REAL
TUNIT4 = ’DEGREE ’ / physical unit of field
TTYPE5 = ’PARMS ’ / label for field 5
TFORM5 = ’5E ’ / data format of field: 4-byte REAL
TUNIT5 = ’ ’ / physical unit of field
EXTNAME = ’AIPS CC ’ / name of this binary table extension
EXTVER = 1 / Table version number
ISORTORD= 0 / AIPSish primary sort key column code
NO_PARMS= 5 /
END

REFERENCES

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