Confusion limited surveys with the ngVLA spiral configuration

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***Abstract*—Counts of radio source density on the sky as a function of flux density can be used to constrain models of the cosmic evolution of the populations involved. Deriving these counts by cataloging individual sources is limited to sources with peak brightnesses of 4–5 times the RMS noise. Confusion limited observations can be used to measure the radio source counts down to or well below the noise level. Interpreting the confusion “noise” requires a good understanding of the synthesized dirty beam as it is in units of Jy per dirty beam solid angle. Furthermore, sidelobes in the dirty beam must be very low as the CLEANing must stop well above the “noise” to avoid modifying the pixel statistics and any remaining unCLEANed sidelobes must be well below the confusion noise level. This memo considers the proposed spiral configuration for the middle component of the ngVLA and concludes that a suitably well behaved dirty beam at $4''$ resolution at S band is feasible A rough estimate of the relative sensitivity of the ngVLA for this observation is approximately a factor of 4.5 over the VLA.**

***Index Terms*—interferometry, confusion limited survey, ngVLA**

I. INTRODUCTION

RADIO source counts, the sky density of sources as a function of flux density, can be used with local luminosity functions to constrain the cosmic evolution of the populations observed. Above about 1 mJy, AGNs dominate the radio source population while below this level, actively star forming galaxies are increasingly dominant.

Source counts derived by making deep images and identifying and categorizing individual sources have a number of limitations. The most obvious of these is that for reliable detection of sources, the peak brightness needs to be 4–5 times the RMS noise. This sets the lower limit on the flux density of the counts. Furthermore, the images must be made with sufficient resolution to identify individual sources and avoid blends, AKA confusion. The derived images are brightness limited rather than flux density limited and the corrections needed to convert the statistics of peak brightnesses to flux densities are not well understood. This possibly explains the wide range of results from attempts to use this technique.

At a sufficient sensitivity, multiple sources can be contributing to each pixel in the image; this is referred to as “confusion”. When the thermal noise in the image is at or below the level of this confusion “noise”, the statistics of the source counts can be inferred from the statistics of the pixel statistics[1]. Being able to interpret the confusion statistics puts constraints on the quality of the synthesized dirty beam.

The simultaneous requirements that (1) the rms noise be comparable with the rms confusion and (2) the image not be limited by dynamic range restrict the frequencies and resolutions of confusion-limited images. For example, the VLA is dynamic-range limited at L-band and lower frequencies. S band (2–4 GHz) is the best choice for confusion-limited VLA images because the sources are stronger and the primary beam is larger than at higher frequencies. At S band, only the VLA C configuration gives the right resolution ($\approx 8''$ FWHM) whose rms confusion ($\approx 0.5\,\mu\mathrm{Jy\ beam}^{-1}$) is comparable with the noise after ≈ 100 hours of integration. This is probably the VLA limit; there are few fields on the sky that would not be dynamic-range limited after significantly longer integrations. The rms confusion is roughly proportional to the synthesized beam solid angle, so the ”sweet spot” for the more sensitive ngVLA is probably $4''$ FWHM resolution at S band, and the rms noise achievable could be $\approx 0.1\,\mu\mathrm{Jy\ beam}^{-1}$. About 90% of the star-formation history of the universe is sampled in sources stronger than $\approx 0.2\,\mu$Jy at S band, so reaching this level is an important scientific goal. Thus the top continuum science goal for SKA1-MID is ”Measuring the star-formation history of the universe” using an ultra deep reference survey to detect individual sources as faint as $0.25\,\mu$Jy at 1 GHz, which would take > 2000 hours and require a ”challenging” >65 dB of dynamic range.

This memo considers the ability of the proposed “spiral” configuration for the ngVLA. Tests using simulated data are presented using the Obit package [2] [1].

II. PROPOSED NGVLA SPIRAL LAYOUT

One proposed configuration for the ngVLA has the antennas at distances from the dense core of a km to several 10s of km arranged in a set of spirals. The instantaneous single channel uv coverage of the central core and spirals for a source at 60° declination at 10 cm wavelength (3 GHz) is shown in Figure 1. This figure is shown as gray-scale to emphasize the highly variable density of uv samples. In Fig. 1 Left, the region of the central core is completely burned in. Fig. 1 Right emphasizes the core region and the extended configuration is nearly invisible.

III. CONSTRAINTS ON THE DIRTY BEAM

In any observation sufficiently sensitive to measure the confusion noise there will be many sources well above the noise level. These must be CLEANed in order to remove

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[1] http://www.cv.nrao.edu/∼bcotton/Obit.html

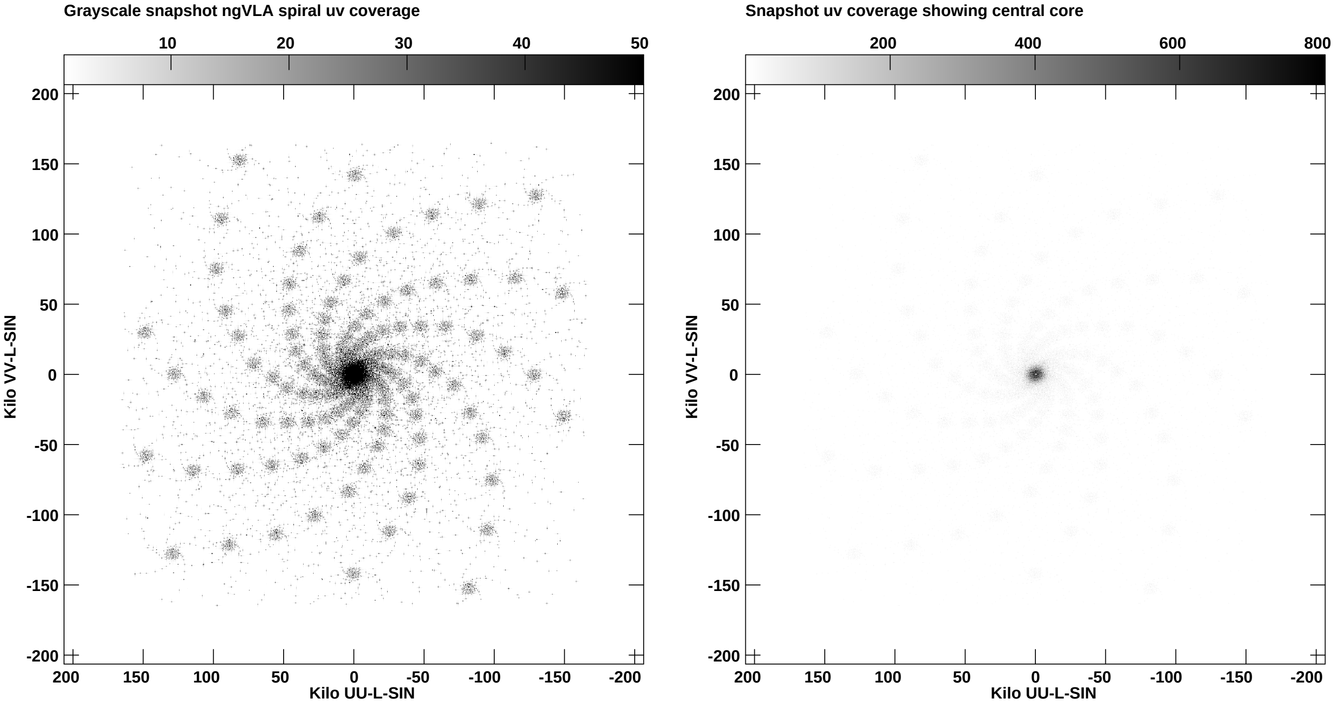

Fig. 1. Left: Snapshot single channel uv coverage of the core and spiral, gray-scale displayed to show the outer portions. Right: same as left but without saturating the inner core. The gray scale shows the natural (u,v)-plane single-channel instantaneous sampling density in units of samples per kilo-lambda squared. Scale bar is given at the top of each plot.

their sidelobes. However, the CLEAN needs to be fairly shallow to avoid modifying the statistics of the confusion; the CLEAN used in [1] stopped at 10 times the RMS. Such a shallow CLEAN will leave unremoved sidelobes which must be well below the noise in order not to disturb the confusion statistics. In practice, the largest acceptable dirty-beam sidelobe amplitudes can be no more than 1%.

A related property of the dirty beam is the "main beam efficiency"; the bulk of the power in the beam should be in the main response and not, say in a pedestal surrounding it. This property is more difficult to evaluate than the sidelobe level.

A final property of the dirty beam is that it should have a well determined beam area as the units of the confusion noise are Janskys per dirty beam area. This is related to the previous property that the bulk of the beam power be in the central response.

IV. SPIRAL CONFIGURATION DIRTY BEAM

In practice, the beam size used for confusion is a compromise between the resolution (and number of independent beam areas in an observation) and sufficient confusion power in a resolution element to be detectable. For a reconfigurable array such as the VLA this can be achieved by choosing the appropriate configuration. For the VLA at S band (2-4 GHz) the $8''$ resolution of the "C" configuration allows the detection of the confusion noise (~ 1 μJy/beam) in several tens of hours.

The ngVLA will not be configurable so a given resolution is obtained by appropriate weighting of the data causing a significant loss of sensitivity. It is anticipated that the ngVLA will have a few times the sensitivity of the VLA (see section V) so a beam FWHM of approximately $4''$ is appropriate.

A. Simulated data and beam

A simulated data-set was generated using the proposed "spiral" configuration for a source at 60° declination, 2048 channels covering the frequency range 2–4 GHz, with samples every 5 min over 6 hours. Obit wide-band imager MFImage using baselines of 0 to 100 kiloλ, a taper of 45 kiloλ, a Briggs' Robust factor of -1.1 (AIPS/Obit usage) using 3×3 uv cells for statistics gives a beam of approximately $4''$with well behaved sidelobes. This beam is shown in Fig. 2 with an azimuthially averaged version in Fig. 3. This beam appears to meet the sidelobe requirements for confusion observations. Generating an acceptable beam involved extensive examination of parameter space. For actual confusion limited observations a more complex weighting scheme will be needed to ensure the same synthesized dirty beam across the wide bandwidth. The weighting used in [1] used a taper and Robust factor per each of the 16 spectral windows to achieve this.

B. Main beam efficiency

Evaluating the main beam efficiency is more difficult than the peak sidelobe levels although Figures 2 and 3 show that the Robust weighting has largely eliminated the pedestal from the central core in the naturally weighted beam. To evaluate the beam power further out in the beam, the cumulative sum of the absolute values of the beam within a given radius was determined and shown in Fig. 4. The figure on the left shows

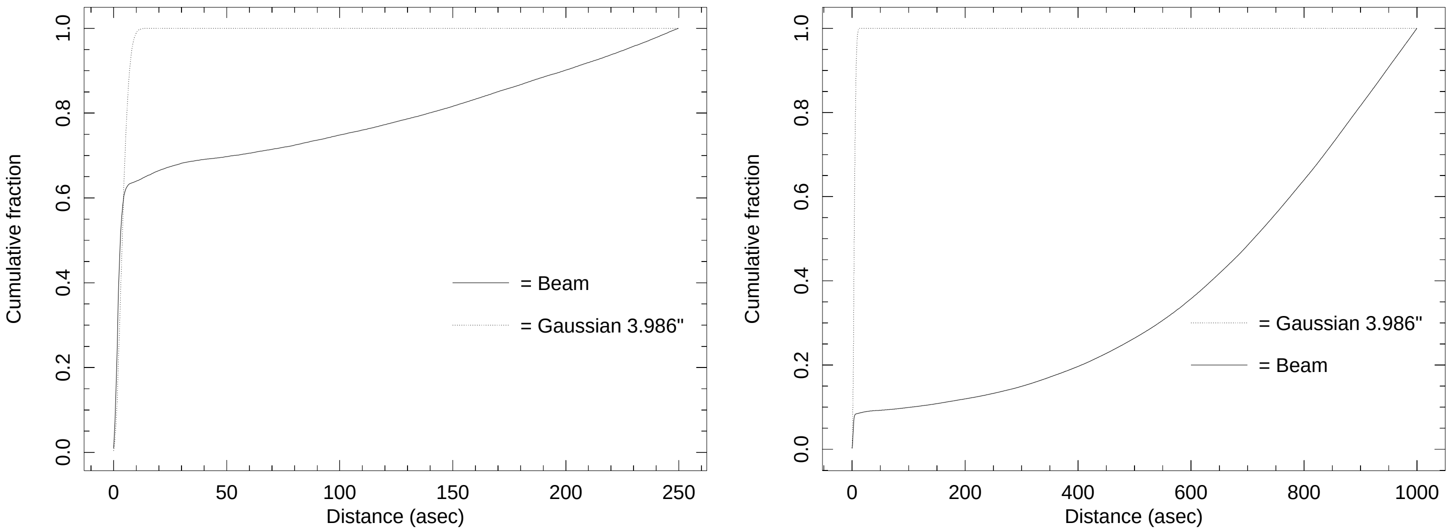

Fig. 4. Left: Cumulative sum of absolute values of beam pixel values within a given distance of the peak. Solid line is from the dirty beam image and the dashed line is an equivalent Gaussian. Right: Like Left but to greater distance.

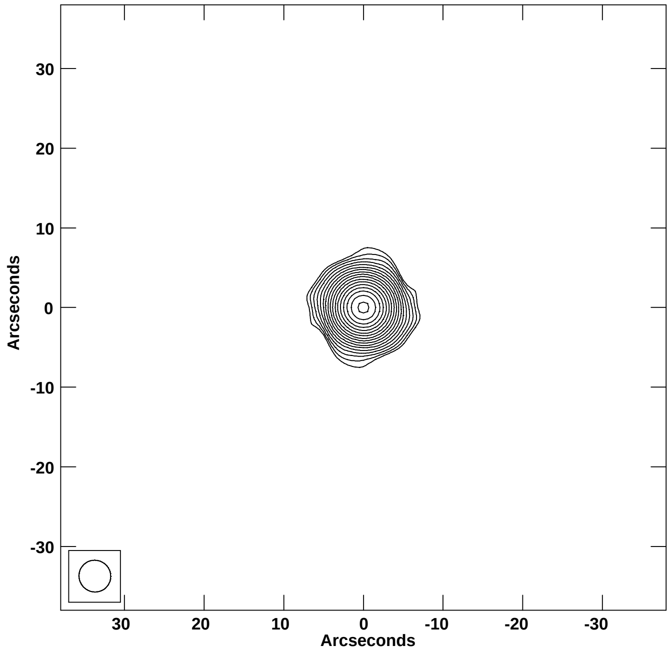

Fig. 2. Contour plot of the $4''$ dirty beam for the ngVLA central core plus spiral configuration. Contours are at every power of $\sqrt{2}$ times 1% from 0.35; negative contours are dashed. Fitted Gaussian is shown in lower left corner.

the power flat from the edge of the beam to approximately 100" and then rising. The right figure traces this trend to greater distance and shows the effect increasing with distance. This effect is not understood but the number of pixels included increases quadratically outward from the center and the use of the absolute value causes the curve to increase monotonically. Since the zero spacing was not measured, the integral of the beam over the entire sky is zero. A second plot was then generated (Fig. 5) using the signed values of the pixels. This plot shows that the beam is biased negative near the peak and then positive at greater distance.

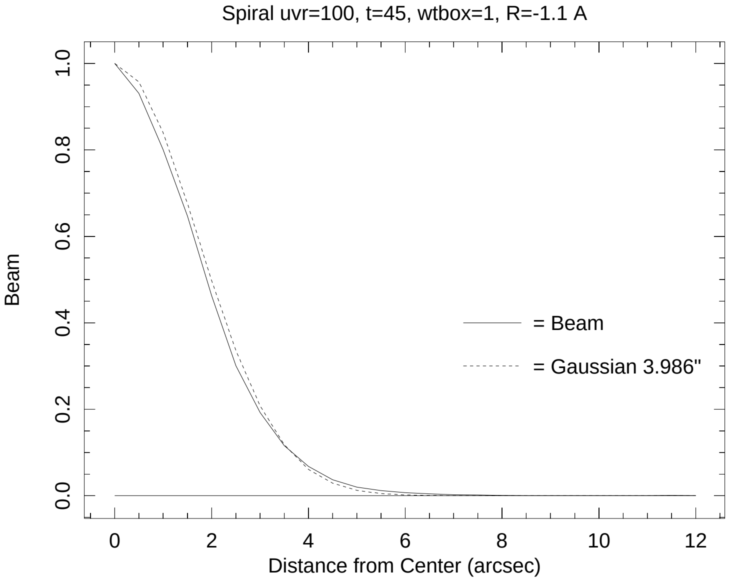

Fig. 3. Azimuthally averaged shape of beam shown in Fig. 2. Solid line shows the beam and the dashed line a Gaussian of the equivalent size.

C. Comparison with VLA

In order to better understand the significance of Fig. 5, a similar data set was generated for a VLA configuration and imaged using MFImage and a Briggs' Robustness of 0. The result is shown in Fig. 6. This plot differs in detail from Fig. 5 but shows the same general features, a negative bias near the peak and a positive bias further out. Since the VLA has been successfully used for confusion observations, this behavior does not appear to present a serious problem.

V. SENSITIVITY WRT VLA

The simulated VLA data described in Section IV-C were generated using the same time and frequency coverage and using the same random noise (in arbitrary units) per baseline/channel; this allows the relative sensitivity of the VLA

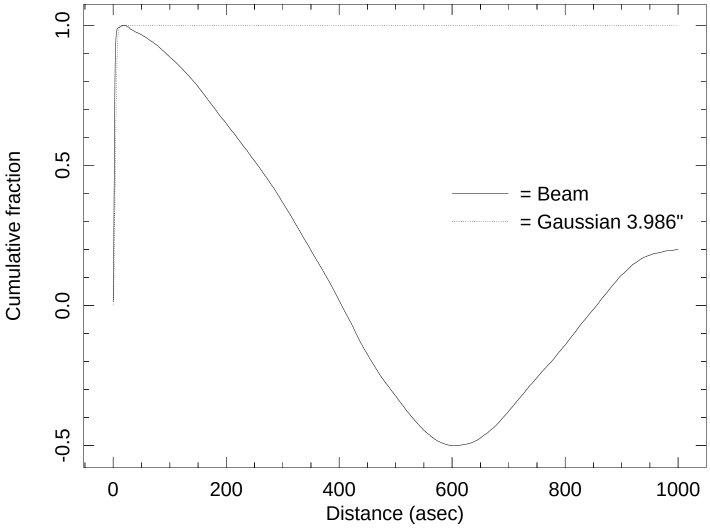

Fig. 5. Like Fig 4 Right but using signed values of the beam.

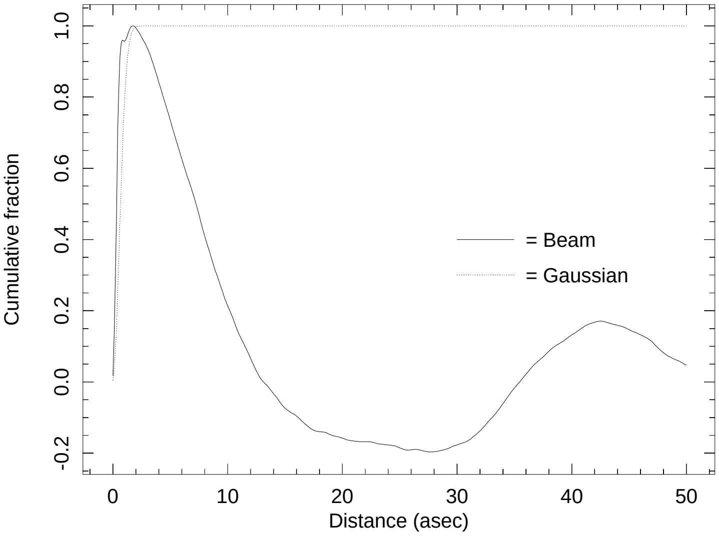

Fig. 6. Like Fig 5 but for a VLA simulation.

and $4''$ resolution ngVLA spiral configuration to be directly compared. The RMS pixel variation in arbitrary units of the ngVLA test was 1.890e-7 and that for the VLA simulation was 7.51e-7. For this test the added noise per baseline/channel was the same in both simulations so the ratio of image RMS needs to be scaled by the ratio of the antenna sensitivities. The expected SEFD for ngVLA at S band is 330 Jy (E. Murphy, private communication) and the achieved value for the VLA is 370 Jy (VLA Observational Status Summary 2018B), thus, the ngVLA antenna should be 12% more sensitive than a VLA antenna making the array sensitivity ratio for this test case 4.5. Note: due to the low resolution of the synthesized beam, many of the ngVLA antennas do not contribute to this result.

VI. CONCLUSIONS

It appears possible to form a beam from the proposed ngVLA “spiral” configuration which is suitable for making confusion measurements with $4''$ resolution at 2-4 GHz. A rough estimate of the relative sensitivity for such an observation is that the ngVLA should be 4.5 times more sensitive than the VLA.

However, not all of the proposed ngVLA S band (1.2–3.5 GHz) may be usable due to problems with RFI and dynamic range. The 18 m antennas will give a large beam on the sky at the lower end of the ngVLA band and it may be difficult or impossible to find a quiet enough celestial position not to have a dynamic range limit above the desired confusion level. Also, the lower end of this frequency range has fierce RFI.

As noted in Section IV-A the actual weighting used to achieve a constant dirty beam in frequency is more complex than that used above to obtain the ngVLA $4''$ resolution beam. The detailed weighting may change the achieved sensitivity.

REFERENCES

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