Single Dish Holography Analysis

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Introduction

- The baseline proposition is to use the CLIC data reduction software of the Plateau de Bure interferometer.

- The raw data should be written in the FITS data format foreseen for the test interferometer (data format prepared by R. Lucas and B. Glendenning).

- A FITS reader will be written at IRAM to convert the data into CLIC internal format.

- The CLIC software needs to be upgraded:
  - to allow for near-field corrections
  - to accomodate both ALMA prototype antennas

Near field error analysis

- The transmitter in the near field region ($R < \frac{D^2}{\lambda}$)

- Allowing for near field transmitter is usually made by including a Fresnel correction, while the Fourier transform is still used (FFT algorithm).

- What are the limits in accuracy in this approximation ?

- Mc Grane (IEE Conf. Pub, No 219.515, 1983) studies the determination of far-field pattern from near field pattern and concludes the method is viable for $R > 0.2\frac{D^2}{\lambda}$.

- Calculations in a draft paper by D. Morris (2000)

Scalar diffraction integral:

$$E_{\mathrm{P}} = \frac{jk}{4\pi} \int \int \frac{E_{\mathrm{M}}}{r} e^{-jkr} [\mathbf{n.r_1} + \mathbf{n.s}] dS$$

Paraxial approximation: $\mathbf{n.r_1} + \mathbf{n.s} = \mathbf{n.r_1} + \mathbf{1}$ is constant, $1/r$ too.
x, y coordinates of M on aperture plane
u, v direction cosines of transmitter P seen from aperture center O
$R = \mathsf{OP}$, $r = \mathsf{MP}$, $r_0 = \mathsf{MP_0}$ (boresight)

$$E(u, v) = \frac{j}{4\pi R} \int A(x, y) e^{-jkr} dS$$

The complex aperture field $A(x, y)$ results from the feed emission diagram, the aperture phase errors, and the refocussing term (displacement from the focus of the paraboloid).

$$r = \sqrt{(Ru - x)^2 + (Rv - y)^2 + R^2(1 - u^2 - v^2)}$$

$$\frac{r}{R} = [1 + \frac{x^2 + y^2}{R^2} - 2\frac{ux + vy}{R}]^{1/2}$$

Approximation:

$$\frac{r}{R} = 1 - \frac{ux + vy}{R} + \frac{x^2 + y^2}{2R^2} + \frac{1}{8}(\frac{x^2 + y^2}{R^2})^2 + \frac{\epsilon}{R}$$

The error term ϵ is:

$$\epsilon = \frac{1}{2R^2}(x^2 + y^2)(ux + vy) - \frac{1}{2R}(ux + vy)^2 + \ldots$$

Thus:

$$E(u, v) = C \int A(x, y) e^{-jk\left[\frac{x^2+y^2}{2R} - \frac{1}{8R^3}(x^2+y^2)^2\right]} e^{jk(ux+vy)} e^{jk\epsilon} dS$$

ϵ is normally ignored to use the FFT.

Upper limit on ϵ:
use extreme values $x_{\mathrm{m}}{=}y_{\mathrm{m}}$, $u_{\mathrm{m}}{=}v_{\mathrm{m}}$, for a circular map:

$$\epsilon_{\mathrm{m}} = \frac{x_{\mathrm{m}}{}^3 u_{\mathrm{m}}}{2R^2} + \frac{x_{\mathrm{m}}{}^2 u_{\mathrm{m}}{}^2}{2R}$$

for a square map:

$$\epsilon_{\mathrm{m}} = \frac{x_{\mathrm{m}}{}^3 u_{\mathrm{m}}}{\sqrt{2}R^2} + \frac{x_{\mathrm{m}}{}^2 u_{\mathrm{m}}{}^2}{R}$$

if no refocussing: $|u_{\mathrm{m}}| = K\frac{D}{2R}$; $|x_{\mathrm{m}}| = D/2$

$$\epsilon_{\mathrm{m}} = \frac{K(\sqrt{2} + 2K)}{32} \cdot \frac{D^4}{R^3}$$

if refocussing: $|u_{\mathrm{m}}| = \frac{sN\lambda}{2D}$; $|x_{\mathrm{m}}| = D/2$

$$\epsilon_{\mathrm{m}} = \frac{DNs\lambda}{32R}[\frac{\sqrt{2}D}{R} + \frac{2Ns\lambda}{D}]$$

Actual calculations (by Dave Morris):
- develop $e^{jk\epsilon}$ term, and use Bessel function for circular data window;
- obtain an integral expression for the path errors, which is integrated numerically;
- the peak, rms, and weighted rms errors are plotted.

$$\epsilon_{\mathrm{m}} =\sim 5166\frac{\pi}{k^2R}(N/128)^2$$

$$\epsilon_{rms} =\sim 812\frac{\pi}{k^2R}(N/128)^{1.6}$$

$$\epsilon_{wrms} =\sim 567\frac{\pi}{k^2R}(N/128)^{1.6}$$

- perform numerical simulations (figure). The errors are concentrated along the edges of the aperture.

Surface error estimates in μm

$D = 12$m, taper 14dB, $\lambda = 3, 25$mm$, R = 300$m.

N	128			256				
	Peak	rms	w. rms	Peak	rms	w. rms		
Sq. w.	28			85			maximum	refocussed
Circ. w.	16			44			maximum	refocussed
Circ. w.	7	1.2	1	28	5	4	formula	refocussed
Sq. w.	3.3	0.5	0.4				simulation	not refocussed
Sq. w.	2.6	1.0	0.9				simulation	refocussed

Conclusions:

- Use FFT seems enough.

- Possibly correct by iterative scheme: compute near field of panel corrections by numerical integration, subtract from measured map, iterate.

- Too big a map (256) may not help.

- Should we make a circular map ?

Data reduction operations

- Convert from Test Interferometer FITS data format to CLIC format. All six products can be stored and later plotted with CLIC vs time, position. Software needs to be written (but of course low level R/W routines for both formats are available).

- Calibrate data in amplitude and phase, based on boresight measurements at regular intervals. A cubic spline is fitted by least squares through data points (Available).

- Interpolate data into a regular grid in antenna-based coordinate system (available)

- pre-FFT corrections:

  - measured reference feed response
  - path difference due to the variation of projected distance between the reference feed and the antenna aperture plane
  - optional amplitude taper to improve PSF

  note: all these corrections merely affect the PSF.

- FFT to aperture plane (64, 128 or 256 points)

Data reduction operations(2)

- Mask edges and blockage (change from PdB to actual antenna blockage)

- Correct for measured phase pattern of feed. Implementation needed (easy subtraction)

- Correct for refocussing path term Δp:

$$\Delta p = \Delta f \cos \alpha - \frac{(\Delta f)^2}{2\rho} \sin^2 \alpha - \frac{x^2 + y^2}{2R} + \frac{(x^2 + y^2)^2}{8R^3}$$

  where $\tan \alpha/2 = \frac{\sqrt{x^2+y^2}}{2f}$ and Δf is the applied focus displacement (96.2mm) that minimizes the integrated square of Δp.

- Least square fit for residual feed position errors, pointing errors, constant phase term (available).

- Convert to normal displacement map: multiply path error by $\frac{\cos \alpha/2}{2}$ (available)

- Fit panel displacements. Include deformations ? (Need: panel geometry).

  CLIC implements an iterative procedure, to allow for finite resolution effects: the fitted panels are used to compute the radiated field which is subtracted from the measured map, the residuals are used to get the next order correction, ... We may use this procedure to correct for near field effects.

Input Data Format for holography reduction

- Use the data format fo the test interferometer, just drafted.

- One main FITS header, plus FITS tables.

- One DATAPAR table for each individual observation (e.g. one map row, one boresight measurement, one arm of a pointing cross, one point of a five-point pointing map, ...). Columns include antenna offset coordinates, flags, ... There is one row for each data point.

- The HOLOPAR should contain for each data point the six receiver-generated products from which the complex field map will computed. The header contains keywords describing the holographic system: frequency, transmitter distance, ...

- Other tables of the data format are not necessarily needed.

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To do List

1. Data format to be reviewed

2. Write FITS to CLIC conversion program.

3. Implement pre-FFT amplitude and phase corrections (reference feed motion, reference feed amplitude and phase diagram, apodisation).

4. Implement post-FFT corrections (feed phase diagram).

5. Get aperture blockage data for both prototypes.

6. Get panel mechanical data for both prototypes, implement in CLIC.

7. Implement near-field corrections in panel fitting loop.