A comparison of the ALMA OTF holography transmitter power to the power radiated at mm wavelengths by a human body

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The ALMA holography transmitter at the OTF radiates about 10 microwatts, into an antenna with a gain of 33 dBi. This radiation is confined to a very narrow bandwidth, of order 1 kHz or less. This radiated power is small compared to the mm-wave thermal radiation from a human body.

The following notes were written and calculated using Mathcad.

**Some physical constants and formulae:**

- Speed of light, m/s: \( c := 2.99792458 \times 10^8 \)
- Planck constant, J.s: \( h := 6.6260755 \times 10^{-34} \)
- Boltzmann constant, J/K: \( k := 1.380658 \times 10^{-23} \)
- Stefan's constant, W/m²/K⁴: \( \sigma := 5.67051 \times 10^{-8} \)

- Assumed radiating surface area of a human body, square meters: \( A := 1 \)
- Assumed temperature of human body: \( T = 300 \text{ K} \)
- Holography frequency: \( f_h := 100 \text{ GHz} \)
- Transmitter antenna gain: \( G := 33 \text{ dBi} \)

**Planck radiation law.**

Watts per Hz at frequency \( f(\text{Hz}) \) and temperature \( T(\text{K}) \):

\[
W_p(f, T) := \frac{2 \cdot A \cdot h \cdot f^3}{c^2 \left( e^{\frac{h \cdot f}{k \cdot T}} - 1 \right)} \cdot \pi
\]

Stefan's law, total power radiated integrated over all frequencies:

\[
W_s(T) := \sigma \cdot A \cdot T^4
\]
Check:

Total Thermal Power radiated: Example of $T := 30$

Comparison Planck with Stefan’s law. At temperature $T=30$, between 0 and $10^15$ Hz:

Planck integration: $\int_0^{10^{15}} W_p(f, T) \, df = 0.04593$ watts

Stefan’s law: $\sigma \cdot A \cdot T^4 = 0.04593$ watts

So the integration of Planck’s law is getting the right answer. This validates the Mathcad Planck equation and integration.

Black-body radiation curves
QUESTION:

What is the 300 K thermal mm-wave power radiated over the spectrum covered by ALMA, from 31 GHz to 950 GHz?

\[
P_{\text{thermal}} := \int_{31 \cdot 10^9}^{950 \cdot 10^9} W_p(f, 300) \, df \quad \text{Pthermal} = 0.078 \text{ watts}
\]

Compare this to the \(10\times10^{-6}\) watts from the transmitter.

\[
P_{\text{tx}} := 10\cdot10^{-6} \text{ watts} \quad \frac{P_{\text{thermal}}}{P_{\text{tx}}} = 7.813 \times 10^3
\]

If we apply the antenna gain of \(G=33\) dB to the transmitter power:

\[
\text{ratio} := \left( \frac{P_{\text{thermal}}}{P_{\text{tx}} \cdot 10^G} \right) \quad \text{ratio} = 3.916
\]

So, even with the transmitter antenna beamed directly at, say, a fuel tank, a person standing at the base of the tower radiates mm-wave radiation \(\sim 4\) times stronger than the holography transmitter.