The Parallel Line Transformer

by

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November, 1998

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Introduction

The parallel line transmission line transformer consists of two sections of transmission line connected in parallel. This arrangement can transform one admittance to another. The simplest example of such a transformer consists of two pieces of transmission line of the same characteristic admittance and same length connected in parallel. (Because the lines are connected in parallel admittances are more appropriate than impedances.) This circuit is easily recognized as equivalent to a transmission line with twice the characteristic admittance (one-half the characteristic impedance) of the individual lines and of the same length as the individual lines.

The parallel line transformer is not restricted to lines with the same characteristic admittance and same length. In the most general case, the lines will have different characteristic admittances and different lengths.

Consider two transmission lines connected in parallel as in Figure 1.

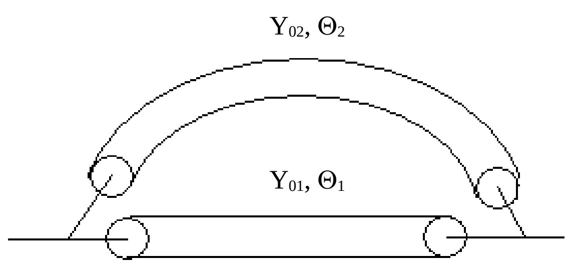

Figure 1. Paralleled Transmission Lines. For convenience coaxial lines are shown and shield connections omitted.

There are at least two different ways of analyzing this arrangement: using network theory to reduce this arrangement to an equivalent two terminal pair network and calculating its characteristic admittance and propagation constant; or, by equating the voltages of the terminals of the two lines where they are connected in parallel, thus determining how the lines share the "load", transforming each line's load through the line and summing the two admittances at the input end.

The Parallel Line Transformer as a Network

The characteristic admittance and propagation constant of a symmetrical two terminal pair network are given by the relations

$$Y_0^2 = Y_{sc} Y_{oc} ,$$

$$\text{Tanh}^2 \gamma = Y_{oc} / Y_{sc} ,$$

where the subscripts sc and oc designate the short and open circuited admittances of the network (one terminal pair shorted and the admittance determined at the other terminal pair, etc.). Because transmission lines and the parallel line connection are symmetric, the short and open circuit admittances are independent of the terminals chosen for measurement and the equivalent networks have only one characteristic admittance.

There is no difficulty in calculating the short circuit admittance of the parallel line connection; it is simply the sum of the short circuit admittances of each line considered independently. The open circuit admittance however is not at all obvious.

Representation as a PI of Susceptances

One way of dealing with this problem is to represent each transmission line as a PI of susceptances. Because the lines are symmetric with respect to their terminals, only two values of susceptance are required: one for the series susceptance and one for the two shunt susceptances. Figure 2 illustrates the PI arrangement.

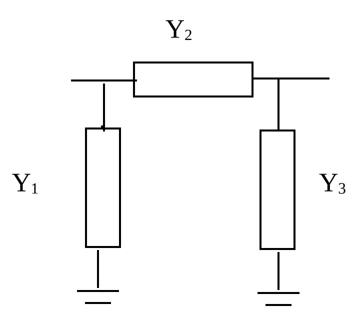

Figure 2. A Transmission Line Represented as a PI of Susceptances.

When two such PI networks are connected in parallel, the susceptances in each position are connected in parallel. The result is a new PI network with

$$Y_{p1} = Y_{11} + Y_{21} ,$$

$$Y_{p2} = Y_{12} = Y_{22} \,,$$

where Y_{pj} represents the parallel combination of Y_{ij} , i identifying the specific line and j designating the position of the susceptance in the PI, series or shunt. The subscript p designates the combination of the two PIs connected in parallel at the two terminals.

For the PI network the short and open circuit admittances are readily seen to be

$$Y_{sc} = Y_1 + Y_2 \,,$$

$$Y_{oc} = (Y_1^2 + 2Y_1 Y_2) / (Y_1 + Y_2).$$

Thus,

$$Y_{0p}^2 = Y_1^2 + 2Y_1Y_2 \,, \qquad (1)$$

$$\mathrm{Tanh}^2\gamma = (Y_1^2 + 2Y_1Y_2)/(Y_1 + Y_2)^2 .$$

Substituting Y_{p1} and Y_{p2} into these relations yields the characteristic admittance and hyperbolic tangent of the propagation constant of the paralleled PIs, and thus of the paralleled transmission lines. These parameters may be used in the usual equation to calculate the admittance change through the paralleled lines.

As a step in the synthesis of a parallel line transformer, the characteristic admittance, Y_0, and propagation constant, γ, required to accomplish the desired transformation can be calculated from the relations

$$Y_0^2 = (G_L|Y_G|^2 - G_G|Y_L|^2)/(G_G - G_L) \,, \qquad (2a)$$

$$\mathrm{Tanh}\gamma = Y_0(G_L - G_G)/(G_GB_L + G_LB_G) \,. \qquad (2b)$$

The subscripts L and G refer to the load and input admittances. Note that the susceptances, B, are imaginary quantities and include the operator j and that 1/j = -j. Note also that Y_0 may be real or imaginary, and that Tanhγ will be imaginary or real accordingly.

The preceding equations for the network parameters can now be solved for Y_{p1} and Y_{p2}. As square roots are involved in these solutions, a total of four solutions are obtained, drawn from the separate solutions for Y_{p1} and Y_{p2}. One pair of these solutions are

$$Y_{p1} = -j[y_{01}\mathrm{ctn}(\theta_1/2) + y_{02}\mathrm{ctn}(\theta_2/2)] \,; \qquad Y_{p2} = j(y_{01}/\sin\theta_1 + y_{02}/\sin\theta_2),$$

and

$$Y_{p1} = j[y_{01}\tan(\theta_1/2) + y_{02}\tan(\theta_2/2)]\ ; \quad Y_{p2} = -j(y_{01}/\sin\theta_1 + y_{02}/\sin\theta_2). \qquad (3)$$

The lower case y's are the magnitudes of the admittances: $y = |Y|$.

These pairs are equivalent and either is a solution to the problem. Depending on how the signs are chosen, another set of "solutions" can be obtained with the central plus signs changed to minus signs. Based on limited numerical exploration these values are not solutions of the original problem and must be discarded.

The above equations can be solved for, say, θ_1 in terms of θ_2:

$$\theta_1 = \sin^{-1}[y_{01}\sin\theta_2/(y_{p2}\sin\theta_2 - y_{02})],$$

$$\theta_1 = 2\mathrm{ctn}^{-1}[-(y_{02}\mathrm{ctn}(\theta_2/2) + y_{p1})/y_{01}]$$

With some additional algebra, the explicit appearance of the PI representation can be removed, leading to the result

$$\theta_1 = 2\tan^{-1}[\tan(\theta_2/2)\{C \pm \sqrt{(C^2 - 4)}\}/2], \qquad (4)$$

$$\theta_1 = \tan^{-1}[y_{01}/\{\sqrt{(-Y_{0p}^2/\mathrm{Tanh}^2\gamma_p)} - y_{02}\mathrm{ctn}\theta_2\}],$$

where

$$C = (Y_{0p}^2 - y_{01}^2 - y_{02}^2)/y_{01}y_{02}$$

and γ_p is the propagation constant of the paralleled transmission lines.

Note that $[C + \sqrt{(C^2 - 4)}]/2 = 2/[C - \sqrt{(C^2 - 4)}]$
.

A Closed-Form Solution

The equations numbered (1), (2) and (3) can be combined with the expression for Y_{p1} in terms of the network parameters,

$$Y_{p1} = -jY_{0p}\mathrm{Tanh}(\gamma_p/2),$$

to produce a closed form solution:

$$j\tan(\theta_2/2) = Y_{0p}\mathrm{Tanh}(\gamma_p/2)/\{y_{01}[C \pm \sqrt{(C^2 - 4)}]/2 + y_{02}\}.$$

Note that either Y_{0p} or $\mathrm{Tanh}(\gamma_p/2)$ will be imaginary.

θ_1 can then, for example, be calculated from θ_2 and Equation (4).

Negative Line Lengths

If one or both of the calculated line lengths is negative, a positive value may be obtained by adding 360 degrees. The addition of 180 degrees to obtain a positive value will produce an incorrect result, as an incorrect sign results for the sine of the line length and this will cause an error in the way the lines divide the load and in the SWR on the individual lines.

Note that going through the transformer in the reverse direction results in reversing the sign of the hyperbolic tangent of the transformer propagation constant which reverses the sign of the line lengths and of the phase shift.

The Range of Application

The solution provides some guidance to the range of characteristic admittances that can be obtained. Clearly the absolute value of C must equal or exceed 2, and this leads to the requirements

$$y_{0p} \geq y_{01} + y_{02} ,$$

or

$$y_{0p} \leq |y_{01} - y_{02}| ,$$

if Y_{0p} is real, and

$$y_{0p} \geq 0$$

if Y_{0p} is imaginary.

Thus, if Y_{0p} is real, the region between $y_{0p} = |y_{01} - y_{02}|$ and $y_{0p} = y_{01} + y_{02}$ is excluded, but any value of y_{0p} is possible if Y_{0p} is imaginary. For the case where the two lines have the same real characteristic admittance, $Y_{0i} = 1$, Y_{0p} is excluded from the region 0 to 2. For $y_{01} = 1$, $y_{02} = 0.6667$ (50 and 75 Ohm lines normalized to 50 Ohms [0.02 and 0.01333 S) the region 0.3333 to 1.6667 is excluded.

As the maximum and minimum values of the sine are 1 and -1, the first expression for θ_1 leads to the inequality

$$y_{02}/(y_{p2} + y_{01}) \geq \sin\theta_2 \geq y_{02}/(y_{p2} - y_{01}).$$

Values of θ_2 outside this range are excluded as possible solutions.

If $|C| < 2$, no solutions to the specific problem exist. If $|C| = 2$, two identical solutions are obtained, and if $|C| > 2$, two different solutions are obtained. When $|C| = 2$, then Y_{0p} will equal $(Y_{01} + Y_{02})$ if Y_{0p} is real or $|Y_{0p}| = |Y_{01} - Y_{02}|$ if Y_{0p} is imaginary.

If the problem of transforming an admittance to 1 +j0 is examined, the required characteristic admittance of the network is given by

$$Y_0^2 = (G_L - |Y_L|^2)/(1 - G_L).$$

This is the equation of a circle in the g-b plane with center at $([1 + Y_0^2]/2 , 0)$ and a radius of $[1 - Y_0^2]/2$. The symbol Y_0 is here complete, so that if it is imaginary its square is negative. In addition to passing through the point (1 , 0), these circles also pass through the point $(Y_0^2 , 0)$.

Figures 3 and 4 show these circles for both real and imaginary characteristic admittances.

For real characteristic admittances, only admittances inside the circle with center (0.5 , 0) and radius 0.5 and to the right of the vertical line g = 1 can be matched to 1 + j0. For imaginary characteristic admittances, only those admittances outside the circle with center at (0.5 , 0) and radius 0.5 and to the left of the vertical line g = 1 can be matched to 1 + j0.

In the more general case, an admittance Y_L can be transformed to an admittance Y_G by a network with a characteristic admittance of Y_0 if both admittances fall on a circle with a center on the G axis of $(Y_0^2 + |Y_L|^2)/2G_L$ and a radius of $\sqrt{((Y_0^2 + |Y_L|^2)^2 - 4G_L^2Y_0^2)}/2G_L$. This circle can be viewed as an "un-normalized" SWR circle if Y_0 is real, or something like an SWR circle if Y_0 is imaginary.

Note that in the g - b plane all admittances that can be attained with a specified imaginary characteristic admittance fall on arcs of circles passing through the points b = ±1. Figure 5 is an example of such a circle diagram. The circular arcs correspond to the SWR circles in the g - b plane for transmission lines where the characteristic admittance is real. These representations of the g - b plane are simplified in that G and B have been normalized not by the imaginary Y_0 but by $|Y_0|$, the magnitude of the characteristic admittance. A complete representation would include the left half of the plane where the normalized conductances, g, would be imaginary and negative. This is of significance only for unsymmetric networks.

Figure 6 shows the range of admittances that can be transformed to 1 + j0 by parallel line transformers made from lines with characteristic admittances of 1 and 0.6667. (Eg, 50 and 75 Ohm coax transforming to 50 Ohms.) Similar diagrams can be made for other line admittances and other input admittances that may be of interest. For example, a 5 mS output admittance (200 Ohms), representing the admittance of a 4:1 balun between 200 and 50 Ohms.

Figure 7 shows the <u>impedances</u> that can be transformed to 200 + j0 Ohms by parallel 300 and 450 Ohm transmission lines. Regions requiring real or imaginary characteristic impedances for matching are labeled. Equations for the circles defining the excluded regions are the same as in admittance coordinates—the center is at $(1 + Z_0^2)/2$ and the radius of the circle is $(1 – Z_0^2)/2$. Similar charts can be prepared for other combinations of transmission lines.

To transform one pure conductance to another pure conductance requires that Y_{0p} be real and equal to $\sqrt{(G_L G_G)}$ and that $\text{Tanh}\gamma_p = j\infty$--that is, a quarter wave transformer. ($\text{Tanh}\gamma = \infty$ with an imaginary characteristic admittance is a possible solution for unsymmetric networks, but not for the parallel line transformer which is symmetric.)

If a section of transmission line is inserted between the load and the parallel line transformer with a characteristic admittance/impedance equal to the design input admittance/impedance effectively any load can be matched to 1+j0. Figure 8 illustrates this arrangement for a transformer of 50 and 75 Ohm lines with a 50 Ohm line section between the load and the transformer. Loads that fall in the excluded regions can be transformed to loads outside these regions by the series line section and the resulting admittance/impedance can be transformed to 1+j0 by the paralleled lines.

If the characteristic admittances of the two "lines" are not assumed to be pure real as above, but to be either pure real or pure imaginary (ie, dissipationless) the solution is

$$\text{Tanh}(\gamma_1/2) = [Y_{0p}\text{Tanh}(\gamma_p/2)]/\{Y_{01} + Y_{02}[C + \sqrt{(C^2 - 4)}]/2\}$$

and

$$\text{Tanh}(\gamma_2/2) = [Y_{0p}\text{Tanh}(\gamma_p/2)]/\{Y_{02} + Y_{01}[C - \sqrt{(C^2 - 4)}]/2\}.$$

Any of the symbols, Y_{ij} or $\text{Tanh}\gamma_i$ may be imaginary, but of course if Y_{0p} is imaginary then $\text{Tanh}\gamma_p$ must be real and the reverse, and similarly for the Y_{0i} and $\text{Tanh}\gamma_i$. Note that the product $Y_{0p}\text{Tanh}\gamma_p/2$ is always imaginary.

This allows analysis when one or both of the "lines" forming the parallel line transformer is replaced with more complex structures. For example, by another parallel line transformer that has an imaginary characteristic admittance. If either Y_{01} or Y_{02} , but not both, is imaginary, then C will be imaginary and C^2 will be negative. In this case, the expression $\sqrt{(C^2 - 4)}$ places no restrictions on what values of Y_{0p} can be attained. If a series line transformer is used as one leg of a parallel line transformer, the parallel line transformer will be unsymmetric. The analysis here does not admit of this case, as the PI representation of the series line transformer requires three, not two, susceptances.

If both Y_{01} and Y_{02} are imaginary, C will be real and the previously described limits apply.

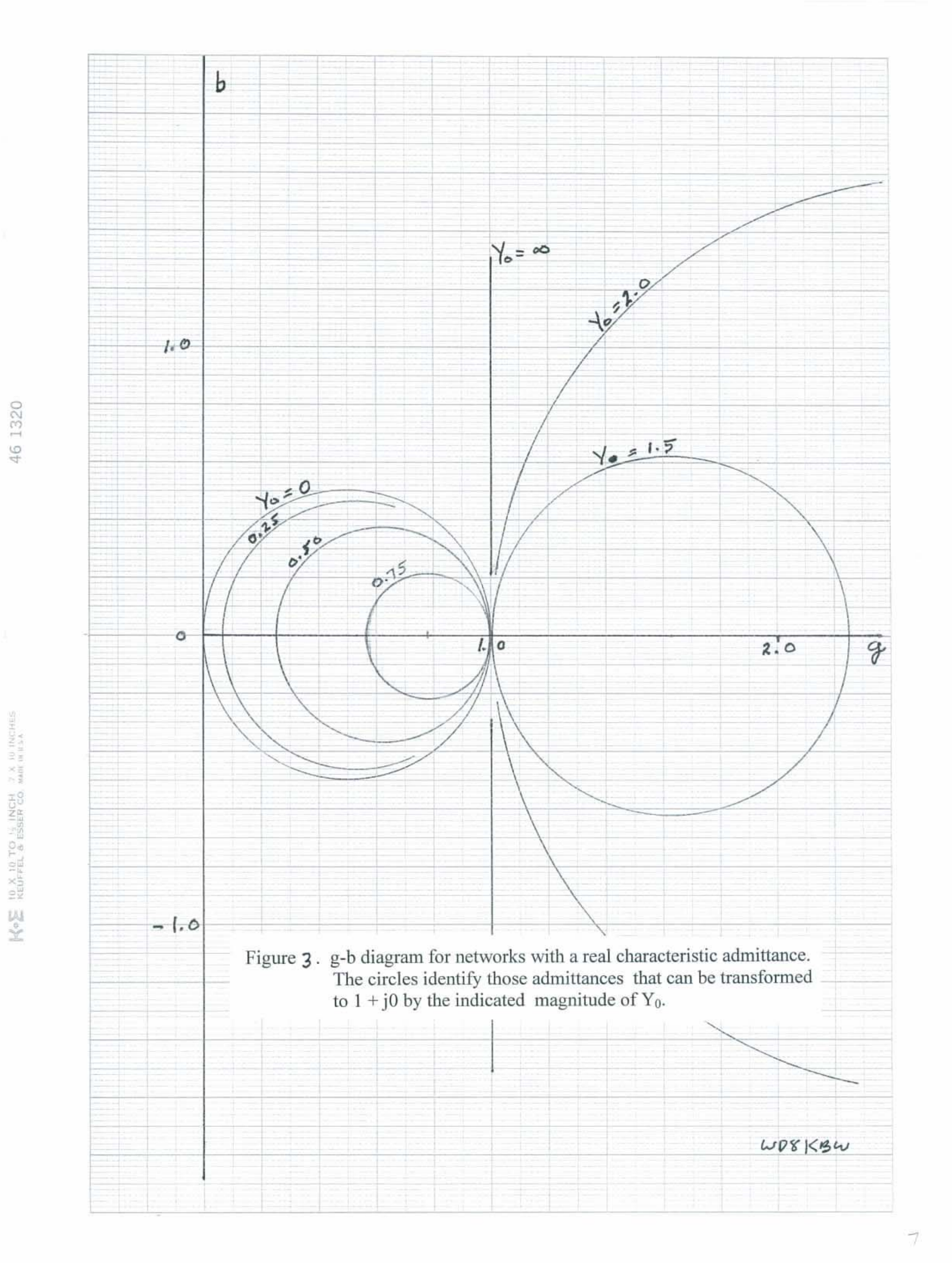

Figure 3. g-b diagram for networks with a real characteristic admittance. The circles identify those admittances that can be transformed to 1 + j0 by the indicated magnitude of Y_0.

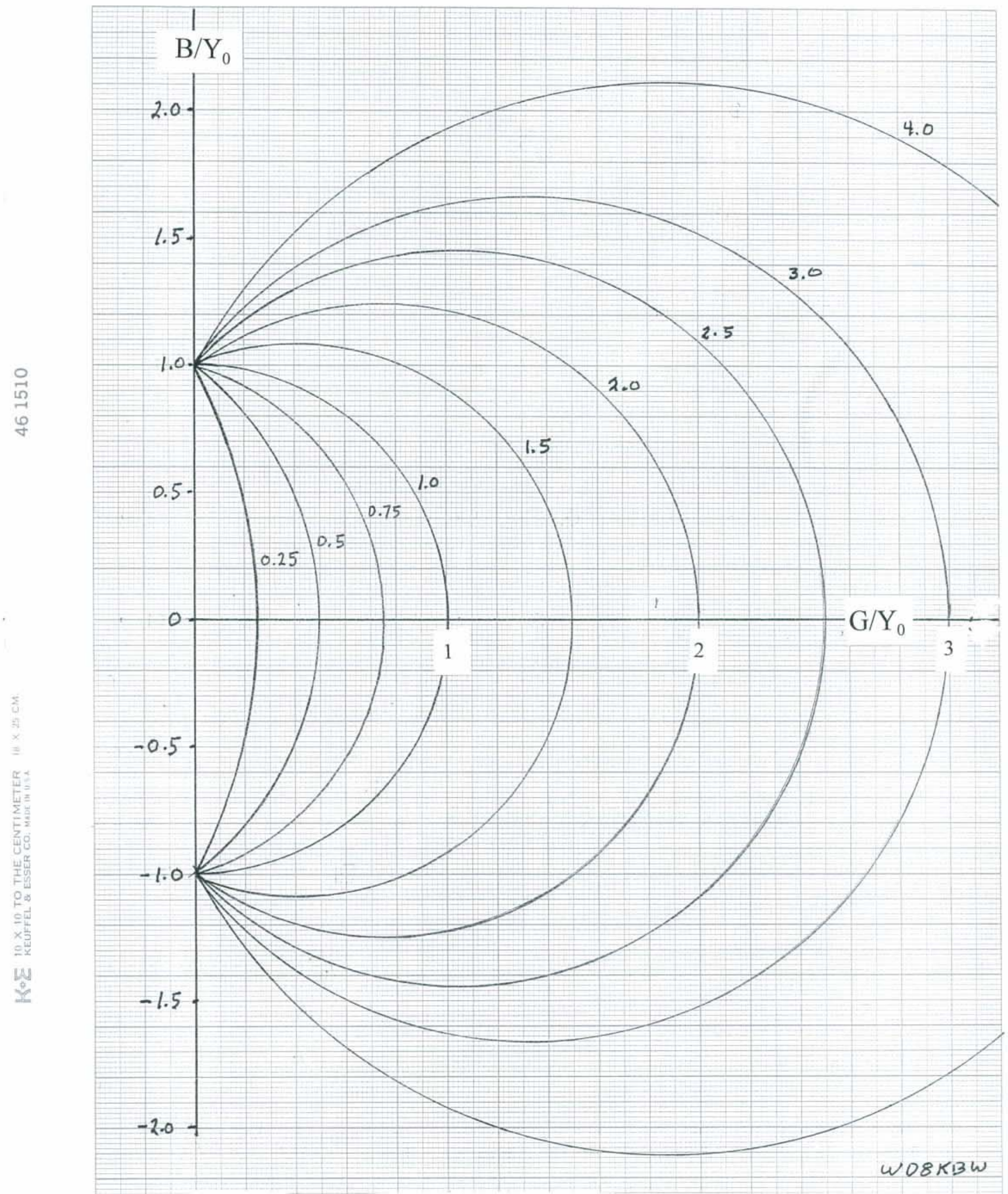

Figure 5. Circle diagram for use with imaginary characteristic admittances.

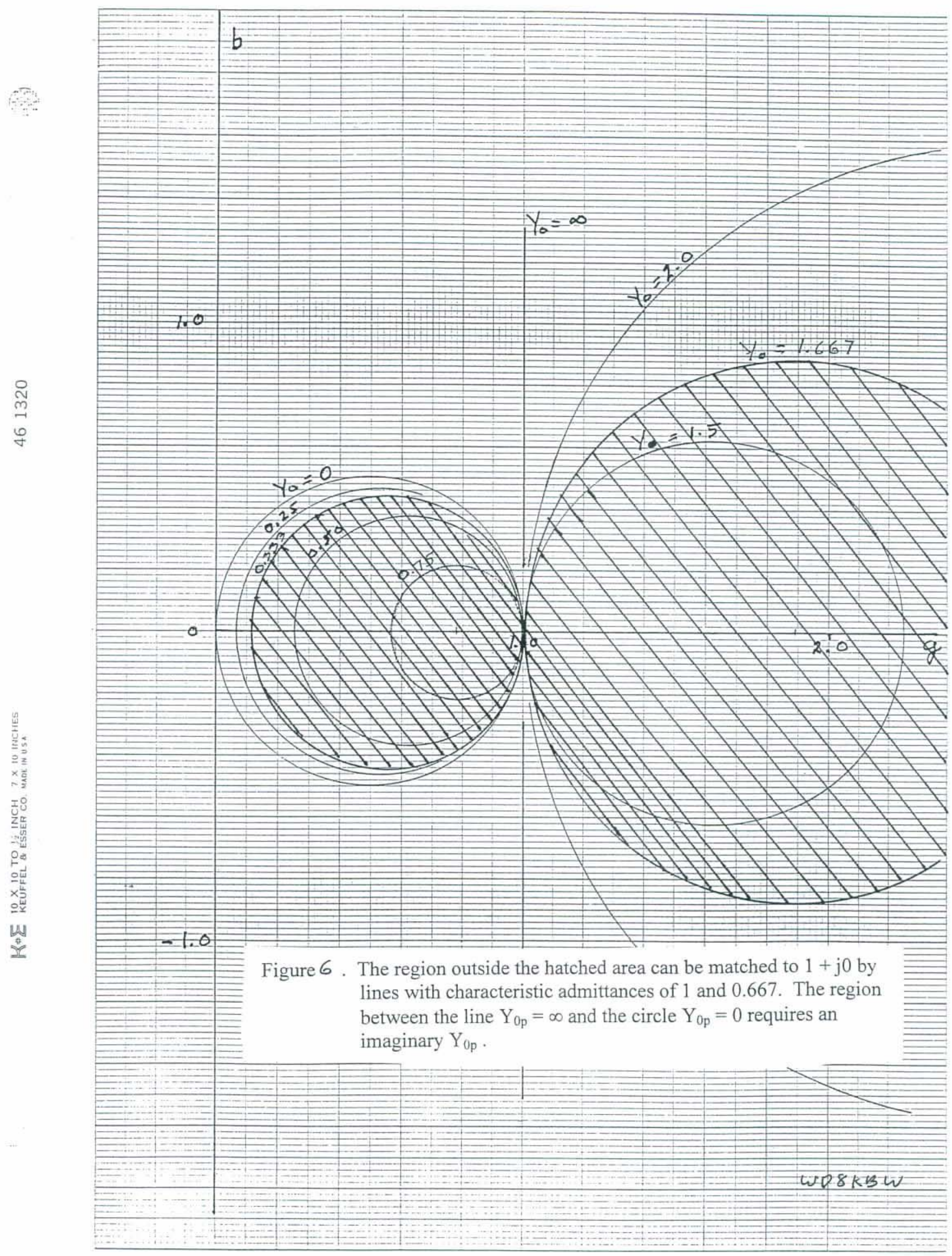

Figure 6 . The region outside the hatched area can be matched to 1 + j0 by lines with characteristic admittances of 1 and 0.667. The region between the line $Y_{0p} = \infty$ and the circle $Y_{0p} = 0$ requires an imaginary Y_{0p} .

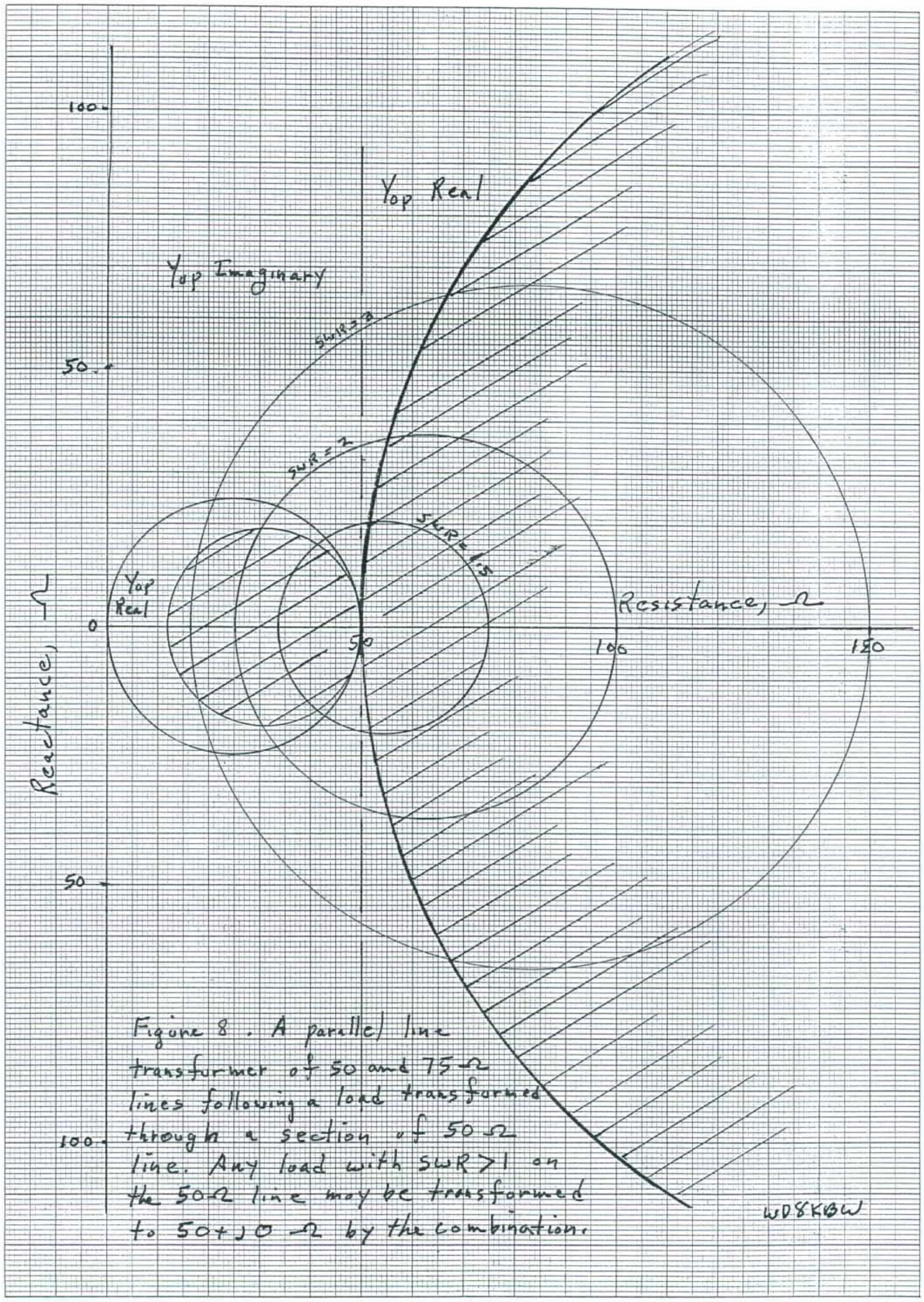

Figure 8. A parallel line transformer of 50 and 75-Ω lines following a load transformed through a section of 50-Ω line. Any load with SWR>1 on the 50-Ω line may be transformed to 50+j0-Ω by the combination.

Another Approach

The second approach to analyzing the parallel line transformer is to equate the line voltages at the line's terminals where they are connected together and from this find how the two lines divide the load. The voltage at the input terminals of a line is given by the relation

$$E_{in} = E_{out}\,(\cos\theta + jY_L \sin\theta / Y_0).$$

Writing such an equation for each of the two lines and equating input and output voltages results in the equation

$$\cos\theta_1 + jY_{L1}\sin\theta_1/Y_{01} = \cos\theta_2 + jY_{L2}\sin\theta_2/Y_{02}\ .$$

Writing the real and imaginary parts separately and noting that the conductance and susceptance of the load is the sum of the values taken by each line, the load admittances taken by the two lines can be found. Thus,

$$G_{1L} = G_L y_{01}\sin\theta_2/(y_{01}\sin\theta_2 + y_{02}\sin\theta_1),$$

$$B_{1L} = [B_L y_{01}\sin\theta_2 - y_{01}y_{02}(\cos\theta_2 - \cos\theta_1)]/\,(y_{01}\sin\theta_2 + y_{02}\sin\theta_1),$$

where G_{1L} and B_{1L} represent the conductance and susceptance seen as a load by Line 1. Similar equations can be written for Line 2. The loads for the two lines can be transformed through the lines and added at the input ends to obtain a solution.

If the problem is to design a parallel line transformer to accomplish a stated transformation, the input admittances of the two lines can be calculated by iterating the line lengths until the input admittances are within some specified error of the required values.

Power Division and Other Parameters of the Individual Lines

More importantly, this approach to the parallel line transformer provides the information to calculate the power carried by each line, the SWR on each line, and the phase shift of the network.

As power is given by the relation E^2G, the lines share the power in the ratios G_{iL}/G_L or the similar ratio at the input end of the lines. Note that the power transmitted by one of the lines may be negative--that is the line is transmitting power from the load to the source. Of course this negative power is supplied by the other line whose transmitted power is greater than the power taken by the load (or supplied by the source). This will occur whenever the sines of the two electrical lengths differ in sign. This requires that one, but not both, line lengths expressed as an angle falls in the third or fourth quadrant.

Circulating power always exists when the characteristic admittance of the transformer is imaginary, but this is not a necessary condition.

The phase shift of the parallel line transformer is calculated as for any two terminal pair network as

$$\tan\varphi = -g_{1L}\tan\theta_1/(y_{01} - b_{1L}\tan\theta_1),$$

or the equivalent equation for line 2. (As the lines are in parallel the phase shift must be the same in both lines.) Note that reversing the sign of the line length reverses the sign of the phase shift. Reversing the sign of the line length is equivalent to passing through the line in the reverse direction.

Computer Codes Solving for the Line Length

The Basic code PARLIN11 is a synthesis code that calculates the line lengths to accomplish a given transformation if the characteristic admittances of the two lines are specified. The Basic code PARSOLV1 is an analysis code that solves for the input admittance given a load admittance when the characteristics of the parallel line transformer-- the line characteristic admittances and the line lengths—are specified.

```
10 CLS
20 PRINT "                         PARLIN11"
30 PRINT "Solution to the Parallel Line Transmission Line Transformer"
45 PRINT "                            by"
50 PRINT "                   Albert E Weller, WD8KBW"
60 PRINT : PRINT
70 GOTO 160
80 'FOR TEST CASES DISABLE LINE 70 and choose one of lines 90-145
'90 GL = 1: BL = 1: GG = 3.01118: BG = 3.37518: YO1=1: YO2=1.5
'100 GL = 1: BL = 1: GG = 6: BG = -6: YO1 = 1: YO2 = 1.5
'110 GL = 1: BL = 1: GG = .6: BG = 1: YO1 = 1: YO2 = 1
'120 GL=.6: BL=.8: GG=1: BG=0: YO1=1: YO2=.6667
'130 GL = 4: BL = 0: GG = 1: BG = 0: YO1 = 1: YO2 = .6667
'140 GL=1.47058802:BL=.7843137:GG=1:BG=0:YO1=1:YO2=2/3
'145 GL=.9008:BL=-.26987:GG=1:BG=0:YO1=1:YO2=1.5
150 GOTO 260
160 PRINT "Enter admittance of load.": PRINT
170 INPUT "     Conductance of load? ", GL
180 INPUT "     Susceptance of load? ", BL: PRINT
190 PRINT "Enter admittance at input of parallel lines": PRINT
200 INPUT "     Conductance at input? ", GG
210 INPUT "     Susceptance at input? ", BG: PRINT
220 PRINT "Enter the characteristic admittances of the two transmission
lines"
230 PRINT "used in the parallel line transformer.": PRINT
240 INPUT "     Characteristic admittance of line 1? ", YO1
250 INPUT "     Characteristic admittance of line 2? ", YO2
260 PI = 3.1415927#
270 'VERSION OF 12/11/97
280 CLS
290 PRINT "For the case YO1 ="; YO1; " :YO2 ="; YO2; " :YL ="; GL;
"+j"; BL; " :and YG="; GG; "+j"; BG
300 PRINT
310 GOSUB 400
320 END
'   Calculating the network parameters to accomplish the desired
'   transformation.
400 YLSQ = GL ^ 2 + BL ^ 2  'magnitude squared of load admittance
410 YGSQ = GG ^ 2 + BG ^ 2  'magnitude squared of admittance at input
420 YOPSQ = (GG * YLSQ - GL * YGSQ) / (GL - GG)  'YOP squared
430 YOP = SQR(ABS(YOPSQ))
440 AA = (YOPSQ - YO1 ^ 2 - YO2 ^ 2) / (YO1 * YO2)
450 IF ABS(AA) < 2 THEN GOTO 1600
460 IF (GG * BL + GL * BG) ^ 2 < .00000001 THEN GOTO 570
465 IF ABS((GL-GG)/(GG*BL+GL*BG))>10E6 THEN GOTO 570
470 TANHPSQ = -YOPSQ * (GL - GG) ^ 2 / (GG * BL + GL * BG) ^
2'hyperbolic tangent of propagation constant
475 IF YOPSQ<0 THEN RR=-1 ELSE RR=1
480 TANHP = (SQR(ABS(TANHPSQ)))*RR*SGN((GL-GG)/(GG*BL+GL*BG))
490 IF TANHPSQ < 0 THEN GOTO 535
500 GAMMAP = .5 * LOG((1 + TANHP) / (1 - TANHP))'ARC HYPERBOLIC TANGENT
510 GAMMAPH = .5 * GAMMAP
520 TANHPH = (EXP(GAMMAPH) - EXP(-GAMMAPH)) / (EXP(GAMMAPH) + EXP(-
GAMMAPH))'HYPERBOLIC TANGENT
530 GOTO 580
535 TANHP=-TANHP
540 GAMMAP = ATN(TANHP)
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```
550 GAMMAPH = .5 * GAMMAP
560 TANHPH = TAN(GAMMAPH): GOTO 580
570 TANHPH = 1: JJ = 3: TANHPSQ = 1E+12 * SGN(GL - GG): GOTO 630
580 IF YOPSQ < 0 THEN JJ = 2
590 IF YOPSQ > 0 THEN JJ = 1
630 MM = (SQR(ABS(YOPSQ))) * TANHPH
640 LL = AA + SQR(AA ^ 2 - 4)
700 GOSUB 800
710 GOSUB 1400
720 LL = AA - SQR(AA ^ 2 - 4)
730 GOSUB 800
740 GOSUB 1400
750 PRINT "Phase shift is "; PHI; "degrees"
760 GOTO 1610
'   Calculating the two line lengths.
800 THETA2 = 2 * ATN(MM / (YO1 * LL/2 + YO2))
810 THETA1 = 2 * ATN(MM / (YO1 + YO2 * 2/LL))
'   Calculating how the two lines divide the load.
900 A = YO1 * SIN(THETA2) + YO2 * SIN(THETA1)
902 E = YO2*TAN(THETA1)
904 W = A*E
910 GL1 = YO1 * SIN(THETA2) * GL / A 'conductance of load presented to
line 1
920 GL2 = YO2 * SIN(THETA1) * GL / A
930 BB = (YO1 * YO2 * (COS(THETA2) - COS(THETA1))) / A    'a constant
940 BL1 = (BL * GL1 / GL) - BB  'susceptance of load presented to line 1
950 BL2 = (BL * GL2 / GL) + BB
'   Calculating phase shift of the network and the SWR on each line.
960 PHI = -57.2958 * ATN((GL1 * TAN(THETA1)) / (YO1 - BL1 *
TAN(THETA1)))
970 PHI = (CINT(PHI * 100)) / 100   'phase shift of the parallel line
network
980 H = (GL1 ^ 2 + BL1 ^ 2 + YO1 ^ 2)
990 SWR1 = (H + SQR(H ^ 2 - 4 * GL1 ^ 2 * YO1 ^ 2)) / (2 * ABS(GL1) *
YO1)
1000 H = (GL - GL1) ^ 2 + (BL - BL1) ^ 2 + YO2 ^ 2
1010 SWR2 = (H + SQR(H ^ 2 - 4 * GL2 ^ 2 * YO2 ^ 2)) / (2 * ABS(GL -
GL1) * YO2)
'   Calculating the sensitivity of the solution to errors in the line
'   lengths--change in achieved YG for 1 percent error in lenghts.
1100 PP = 1.01: QQ = 1
1110 GOSUB 1170
1120 REERR1 = REERR: IMERR1 = IMERR
1130 PP = 1: QQ = 1.01
1140 GOSUB 1170
1150 REERR2 = REERR: IMERR2 = IMERR
1160 GOTO 1340
1170 T1 = TAN(PP * THETA1)
1180 DEN1 = (YO1 - BL1 * T1) ^ 2 + GL1 ^ 2 * T1 ^ 2
1190 REYG1 = YO1 ^ 2 * GL1 * (1 + T1 ^ 2) / DEN1
1200 IMYG1 = (-YO1 * T1 * (GL1 ^ 2 + BL1 ^ 2 - YO1 ^ 2) + BL1 * YO1 ^ 2
* (1 - T1 ^ 2)) / DEN1
1210 T2 = TAN(QQ * THETA2)
1220 DEN2 = (YO2 - BL2 * T2) ^ 2 + GL2 ^ 2 * T2 ^ 2
1230 REYG2 = YO2 ^ 2 * GL2 * (1 + T2 ^ 2) / DEN2
1240 IMYG2 = (-YO2 * T2 * (GL2 ^ 2 + BL2 ^ 2 - YO2 ^ 2) + BL2 * YO2 ^ 2
* (1 - T2 ^ 2)) / DEN2
```

```
1250 REYG = REYG1 + REYG2: IMYG = IMYG1 + IMYG2
1260 YGESQ = (REYG1 + REYG2) ^ 2 + (IMYG1 + IMYG2) ^ 2
'1270 REERR = (GG*REYG + BG*IMYG)/YGESQ  'multiplicative/divisive
sensitivity
1280 REERR = REYG - GG                   'Additive/subtractive
sensitivity
'1290 IMERR = (BG*REYG - IMYG*GG)/YGESQ  'multiplicative/divisive
sensitivity
1300 IMERR = IMYG - BG                   'Additive/subtractive
sensitivity
1310 REERR = (CINT(REERR * 100)) / 100
1320 IMERR = (CINT(IMERR * 100)) / 100
1330 RETURN
1340 RETURN
1400 PRINT "  THETA1"; " THETA2"; "    SWR1"; "    SWR2"
'1410 IF THETA1 < 0 THEN THETA1 = THETA1 + 2 * PI
'1420 IF THETA2 < 0 THEN THETA2 = THETA2 + 2 * PI
1430 PRINT USING "####.## "; THETA1 * 180 / PI; THETA2 * 180 / PI;
SWR1; SWR2
1440 PRINT "Fraction of power in lines 1 and 2:"
1450 PRINT USING "  ##.##  "; GL1 / GL; GL2 / GL
1460 PRINT "Sensitivity is "; REERR1; "+ j"; IMERR1; "for one percent
error in"
1470 PRINT "     theta1 and "; REERR2; "+j"; IMERR2; "for one percent
error in theta2"
1480 PRINT
1490 RETURN
1600 PRINT "No solution is possible": PRINT
1610 IF JJ = 1 THEN GOTO 1650
1620 IF JJ = 2 THEN GOTO 1660
1630 IF JJ = 3 THEN GOTO 1640
1640 PRINT "Network parameters are YOP="; YOP; "  TANHP=jTAN(90
DEGREES)": GOTO 1670
1650 PRINT "Network parameters are YOP="; YOP; "  TANHP=j"; TANHP: GOTO
1670
1660 PRINT "Network parameters are YOP=j"; YOP; "  TANHP="; TANHP: GOTO
1670
1670 RETURN
```

```
0 CLS
5 PRINT "                       PARSOVL1":PRINT
10 PRINT " SOLVES PARALLEL LINES GIVEN THETA1 AND THETA2, OUTPUTS INPUT
ADMITTANCE"
15 PRINT "                  by A. E. Weller, WD8KBW": PRINT
30 INPUT "ENTER LOAD ADMITTANCE, G AND B.  ", GL, BL
40 INPUT "ENTER YO1, YO2.  " ,YO1, YO2
50 INPUT "ENTER THETA1, THETA2 IN DEGREES.  ",THETA1, THETA2: PRINT
60 'VERSION OF 12/11/97
90 PI = 3.141593
110 THETA1 = THETA1 * PI / 180: THETA2 = THETA2 * PI / 180
120 A = YO1 * SIN(theta2) + YO2 * SIN(THETA1)
'125 PRINT A
130 GL1 = YO1 * SIN(theta2) * GL / A
140 GL2 = YO2 * SIN(THETA1) * GL / A
```

```
150 B = (YO1 * YO2 * (COS(theta2) - COS(THETA1))) / A
'155 PRINT B
160 BL1 = (BL * GL1 / GL) - B
170 bl2 = (BL * GL2 / GL) + B
180 C = (YO1 - BL1 * TAN(THETA1)) ^ 2 + GL1 ^ 2 * (TAN(THETA1)) ^ 2
190 GG1 = (YO1 ^ 2 * GL1 * (1 + (TAN(THETA1)) ^ 2)) / C
200 BG1 = (YO1 * (-(GL1 ^ 2 + BL1 ^ 2 - YO1 ^ 2)) * TAN(THETA1) + BL1 *
YO1 ^ 2 * (1 - (TAN(THETA1)) ^ 2)) / C
210 C = (YO2 - bl2 * TAN(theta2)) ^ 2 + GL2 ^ 2 * (TAN(theta2)) ^ 2
220 GG2 = (YO2 ^ 2 * GL2 * (1 + (TAN(theta2)) ^ 2)) / C
230 BG2 = (YO2 * (-(GL2 ^ 2 + bl2 ^ 2 - YO2 ^ 2)) * TAN(theta2) + bl2 *
YO2 ^ 2 * (1 - (TAN(theta2)) ^ 2)) / C
240 GG = GG1 + GG2: BG = BG1 + BG2
250 PHI = -57.2958 * ATN((GL1 * TAN(THETA1)) / (YO1 - BL1 *
TAN(THETA1)))
260 PHI = (CINT(PHI * 100)) / 100   'phase shift of the parallel line
network
270 H = (GL1 ^ 2 + BL1 ^ 2 + YO1 ^ 2) 'version of 11/20/97
280 SWR1 = (H + SQR(H ^ 2 - 4 * GL1 ^ 2 * YO1 ^ 2)) / (2 * ABS(GL1) *
YO1)
290 H = (GL - GL1) ^ 2 + (BL - BL1) ^ 2 + YO2 ^ 2
300 SWR2 = (H + SQR(H ^ 2 - 4 * GL2 ^ 2 * YO2 ^ 2)) / (2 * ABS(GL -
GL1) * YO2)
310 CLS
320 PRINT "For
theta1=";THETA1*180/PI;",theta2=";theta2*180/PI;",YO1=";YO1;
",YO2=";YO2;",GL=";GL;",BL=";BL
330 PRINT "     GG"; "            BG";"          SWR1";"           SWR2"
340 PRINT GG, BG, SWR1, SWR2
350 PRINT "POWER DIVIDISION BETWEEN LINES 1 AND 2 IS"; GL1/GL;"/";GL2/GL
360 PRINT "PHASE SHIFT IS"; PHI; "DEGREES"
370 END
```

Notes for PARLIN11

GL	conductance of load
BL	susceptance of load
YL	admittance of load
YLSQ	square of the magnitude of the load admittance
GG	conductance at input
BG	susceptance at input
YG	admittance at input
YGSQ	square of the magnitude of the admittance at the input
YO1	characteristic admittance of line 1
YO2	characteristic admittance of line 2
YOP	characteristic admittance of the parallel line xfmr
YOPSQ	square of YOP
TANHP	hyperbolic tangent of the propagation constant of the parallel line xfmr
TANHPSQ	square of TANHP
GAMMAP	propagation constant of the parallel line xfmr
GAMMAPH	GAMMAP/2
TANHPH	hyperbolic tangent of GAMMAPH
GL1	admittance at load end of line 1
BL1	susceptance at load end of line 1
GL2	admittance at load end of line 2
BL2	susceptance at load end of line 2
THETA1	electrical length of line 1, radians or degrees
THETA2	electrical length of line 2, radians or degrees
SWR1	SWR on line 1
SWR2	SWR on line 2
REERRi	real part of error in YG for a 1 percent error in both THETAi
IMERRi	imaginary part of error in YG for a 1 percent error in both THETAi
DEN1	denominator 1, line 1
DEN2	denominator 2, line 2
REYGk	real part of YGk if THETAi changed by 1%
IMYGk	imaginary part of YGk if THETAi changed by 1%

```
THETA1=2*ATN(YOP*TANPH/(YO1 + AA*YO2))

THETA2=2*ATN(YOP*TANPH/(YO1/AA + YO2))

AA=(C+/-SQR(C^2-4))/2

C=(YOP^2-YO1^2-Y02^2)/YO1*YO2
```

If C<2, there are no solutions
If C=2, there are two identical solutions*
If C>2, there are two different solutions

* If C=2, then YOP=YO1+YO2 if YOP is real, or j|YO1-YO2| if YOP is imaginary, and THETA1 will equal THETA2.

The sensitivity of the solution to small errors in the lengths of the two lines is given as either YG/(YG with +1 percent errors in the lengths) or as (YG with a +1 percent error in the lengths) minus the correct value of YG. You may select the representation you prefer.

Power on one line greater than one and negative on the other line indicates circulating power. In general this is undesirable as it will increase losses as compared to lines without circulating power and the same SWR. All solutions having an imaginary Y0p have circulating power, but this is not a necessary criterion.

Solutions (rounded) for the test problems are:

LINE NUMBER	THEA1	THEA2	SWR1	SWR2	PWR1	PWR2	PHI	YOP	TANHP
90	60	30	3.60	3.22	0.28	0.72	-24.59	2.68	j0.844
100	139.87	51.66	3.62	6.58	0.45	0.55	45	3.46	j∞
110	6.67	347.60	4.97	3.26	2.18	-1.18	-18.43	j0.632	-0.158
120	335.74	71.83	5.33	8.16	1.41	-0.41	12.09	j1.0	0.5
130	119.59	50.73	2.44	2.88	0.57	0.43	-90	2.0	j∞
140	314.99	315.02	1.67	1.67	0.60	0.40	30.96	1.67	j1.0
145	3.89	355.08	5.37	4.24	-5.36	6.36	20.18	0.41	j0.150
145	18.64	345.22	1.19	1.44	-1.14	2.14	20.18	0.41	J0.150

Note the high sensitivity of the solution to the test problem on line 140. This would not be a practical solution as the lines could not be cut accurately enough to insure a reasonable error. One percent error, as used in the code, would be about 6 inches for a 100 degree length of cable with a velocity factor of 0.67 at 3.5 MHz, 0.75 inch at 29 MHz, and only about 0.15 inches at 146 MHz. An additive/subtractive error of perhaps 0.05/0.1 +/- j0.05/0.1 may be acceptable for a usable parallel line xfmr.

Ideal solutions would have line lengths nearly equal, good division of Power, low SWR, short total line length, and a low sensitivity to errors in the line lengths.

Your comments are welcome.