Appendix B

The Relationship Between Flux and Brightness Temperature

In the following I derive the relationship between the flux of a source and its brightness temperature for a gaussian beam measuring two different source geometries, uniform disk and gaussian. The general relation between flux and brightness temperature is

$$S_\nu = \frac{2k}{\lambda^2} \int T_B(\Omega) d\Omega \tag{B.1}$$

Note that throughout this discussion when I refer to a "temperature" I am actually referring to the effective source radiation temperature $J(\nu, T)$, which is defined by Equation C.1.

B.1 Uniform Disk Source

$$\begin{aligned} S_\nu &= \frac{2k}{\lambda^2} \int_0^{R_p} 2\pi r \left(T_p - T_{bg}\right) \left\{ \exp\left[-4\ln(2) \left(\frac{r}{\theta_B}\right)^2 \right] \right\} dr \\ &= \frac{2k(T_p - T_{bg})}{\lambda^2} \frac{\pi \theta_B^2}{4\ln(2)} \left\{ 1 - \exp\left[-\ln(2) \left(\frac{\theta_{eq}\theta_{pol}}{\theta_B^2} \right) \right] \right\} \end{aligned} \tag{B.2}$$

where I have used the fact that

$$\frac{d}{dr}\left\{ \exp\left[-4\ln(2) \left(\frac{r}{\theta_B}\right)^2 \right] \right\} = -\frac{8\ln(2) r}{\theta_B^2} \left\{ \exp\left[-4\ln(2) \left(\frac{r}{\theta_B}\right)^2 \right] \right\} \tag{B.3}$$

$$\sqrt{\theta_{eq}\theta_{pol}} = 2R_p \tag{B.4}$$

$$\tag{B.5}$$

Plugging in some constants into Equation B.2 yields the following

$$
\begin{aligned}
S_\nu(Jy) &= \frac{2k}{c^2}10^{23}\left[\frac{(2\pi)(10^9)}{(360)(3600)}\right]^2 \frac{\pi\nu^2(GHz)\theta_B^2(arcsec)}{4\ln(2)}\left\{1-\exp\left[-\ln(2)\left(\frac{\theta_{eq}\theta_{pol}}{\theta_B^2}\right)\right]\right\}(T_p - T_{bg})(K) \\
&= 8.179\times 10^{-7}\nu^2(GHz)\theta_B^2(arcsec)\left\{1-\exp\left[-\ln(2)\left(\frac{\theta_{eq}\theta_{pol}}{\theta_B^2}\right)\right]\right\}(T_p - T_{bg})(K) \qquad \text{(B.6)}
\end{aligned}
$$

The peak flux densities of the planets are calculated by the `planets` program using Equation B.6.

B.2 Elliptical Gaussian Source

$$
\begin{aligned}
S_\nu &= \frac{2k}{\lambda^2}\iint_{-\infty}^{\infty}(T_p - T_{bg})\left\{\exp\left[-4\ln(2)\left(\frac{r^2}{\theta_B^2+\theta_{min}^2+\theta_{max}^2}\right)\right]\right\}d^2r \\
&= \frac{2k(T_p-T_{bg})}{\lambda^2}\frac{4\pi}{16\ln(2)}\left[\theta_B^2+\theta_{min}^2+\theta_{max}^2\right] \\
&= \frac{k\pi\theta_B^2(T_p-T_{bg})}{2\ln(2)\lambda^2}\left[1+\frac{\theta_{min}^2+\theta_{max}^2}{\theta_B^2}\right] \qquad \text{(B.7)}
\end{aligned}
$$

Plugging in some constants into Equation B.7 yields the following

$$
\begin{aligned}
S_\nu(Jy) &= \frac{2k}{c^2}10^{23}\left[\frac{(2\pi)(10^9)}{(360)(3600)}\right]^2 \frac{\pi\nu^2(GHz)\theta_B^2(arcsec)}{4\ln(2)}\left\{1+\frac{\theta_{min}^2+\theta_{max}^2}{\theta_B^2}\right\}(T_p - T_{bg})(K) \\
&= 8.179\times 10^{-7}\nu^2(GHz)\theta_B^2(arcsec)\left\{1+\frac{\theta_{min}^2+\theta_{max}^2}{\theta_B^2}\right\}(T_p - T_{bg})(K) \qquad \text{(B.8)}
\end{aligned}
$$