# Appendix C

# Temperature Scales and Telescope Efficiencies

The calibration mode used for essentially all spectral line observations at the 12m is the chopper wheel method (see Ulich & Haas, 1976, ApJS, 30, 247 for a detailed description of this technique). The chopper wheel technique corrects for atmospheric attenuation and several telescope losses. In the following, I describe the temperature scale used at the 12m and how it relates to temperature scales used at other millimeter wavelength observatories.

## C.1 Definitions

In the following I define the terms used in the subsequent temperature scale and telescope efficiency discussion. I have tried to adopt a similar nomenclature to that used in Kutner & Ulich (1981, ApJ, 250, 341). Note that throughout this discussion when I refer to a "temperature" I am actually referring to the effective source radiation temperature $J(\nu, T)$, which is defined as:

$$J(\nu, T) \equiv \left(\frac{h\nu}{k}\right) \frac{1}{\exp\left(\frac{h\nu}{kT}\right) - 1} \tag{C.1}$$

**General Terms**

$\Omega_s \equiv$ Solid angle subtended by the source

$\Omega_d \equiv$ Solid angle subtended by the central diffraction beam pattern of the telescope

$\Omega \equiv$ Solid angle on the sky

$\Psi \equiv$ Direction angle on the sky

$P_n \equiv$ Normalized antenna power pattern

$P_{ng} \equiv$ Normalized Gaussian antenna power pattern

$B_n \equiv$ Normalized source brightness distribution

$A \equiv$ Airmass toward which the measurement is made

$\tau_0$ ≡ Atmospheric optical depth at the zenith
$G$ ≡ Maximum antenna gain

**Efficiencies**

$$\begin{aligned}\eta_r &\equiv \text{Radiative efficiency}\\ &\equiv \frac{G}{4\pi}\iint_{4\pi} P_n(\Omega)d\Omega \end{aligned}\tag{C.2}$$

$$\begin{aligned}\eta_{rss} &\equiv \text{Rearward scattering and spillover efficiency}\\ &\equiv \frac{\iint_{2\pi} P_n(\Omega)d\Omega}{\iint_{4\pi} P_n(\Omega)d\Omega} \end{aligned}\tag{C.3}$$

$$\eta_l \equiv \eta_r\eta_{rss}\tag{C.4}$$

$$\begin{aligned}\eta_{fss} &\equiv \text{Forward scattering and spillover efficiency}\\ &\equiv \frac{\iint_{\Omega_d} P_n(\Omega)d\Omega}{\iint_{2\pi} P_n(\Omega)d\Omega} \end{aligned}\tag{C.5}$$

$$\begin{aligned}\eta_{mb} &\equiv \text{Main beam efficiency}\\ &\equiv \frac{\iint_{4\pi} P_{ng}(\Omega)d\Omega}{\iint_{4\pi} P_n(\Omega)d\Omega} \end{aligned}\tag{C.6}$$

$$\begin{aligned}\eta_{cmb} &\equiv \text{Efficiency at which the source couples to the main diffraction beam}\\ &\quad\ \text{of the telescope}\\ &\equiv \frac{\iint_{\Omega_s} P_n(\Psi-\Omega)B_n(\Psi)d\Psi}{\iint_{4\pi} P_{ng}(\Omega)d\Omega} \end{aligned}\tag{C.7}$$

$$\begin{aligned}\eta_c &\equiv \text{Efficiency at which the source couples to the telescope beam}\\ &\equiv \eta_{cmb}\eta_{mb}\\ &\equiv \frac{\iint_{\Omega_s} P_n(\Psi-\Omega)B_n(\Psi)d\Psi}{\iint_{4\pi} P_n(\Omega)d\Omega} \end{aligned}\tag{C.8}$$

(C.9)

**Temperatures**

$T_R$ ≡ Source radiation temperature

$$\begin{aligned}T_A &\equiv \text{Observed source antenna temperature}\\ &\equiv \frac{GT_R}{4\pi}\exp(-A\tau_0)\iint_{\Omega_s} P_n(\Psi-\Omega)B_n(\Psi)d\Psi \end{aligned}\tag{C.10}$$

$$
\begin{aligned}
T'_A &\equiv \text{Observed source antenna temperature corrected for atmospheric attenuation} \\
&\equiv T_A \exp(A\tau_0) && \text{(C.11)}
\end{aligned}
$$

$$
\begin{aligned}
T^*_A &\equiv \text{Observed source antenna temperature corrected for atmospheric attenuation,} \\
&\quad\ \text{radiative loss, and rearward scattering and spillover} \\
&\equiv \frac{T'_A}{\eta_r \eta_{rss}} && \text{(C.12)}
\end{aligned}
$$

$$
\begin{aligned}
T^*_R &\equiv \text{Observed source antenna temperature corrected for atmospheric attenuation,} \\
&\quad\ \text{radiative loss, and rearward and forward scattering and spillover}^1 \\
&\equiv \frac{T^*_A}{\eta_{fss}} && \text{(C.13)}
\end{aligned}
$$

$$
\begin{aligned}
\Delta T_R &\equiv \text{Source radiation temperature excluding any background emission} \\
&\quad\ \text{(like the cosmic microwave background emission)} \\
&\equiv T_R - T_{bg} \\
&\equiv \frac{T^*_R}{\eta_c} && \text{(C.14)}
\end{aligned}
$$

$$
\begin{aligned}
T_{mb} &\equiv \text{Source brightness temperature as measured by the main diffraction beam} \\
&\quad\ \text{of the telescope} \\
&\equiv \eta_{cmb} \Delta T_R && \text{(C.15)}
\end{aligned}
$$

## C.2 Relations Between Temperature Scales

We can now combine the definitions above to derive the relations between the physical measurements and the temperature scale used at the 12m and the scales used at other telescopes. Combining the equations above, we can relate the source temperature corrected for atmospheric attenuation ($T'_A$) to many of the antenna and source temperatures:

$$
\begin{aligned}
T'_A &= \eta_r \eta_{rss} \eta_{fss} \eta_c \Delta T_R && \text{(C.16)} \\
&= \eta_l \eta_{fss} T^*_R && \text{(C.17)} \\
&= \eta_l T^*_A && \text{(C.18)} \\
&= \eta_{mb} T_{mb} && \text{(C.19)}
\end{aligned}
$$

[1] $T^*_R$ can also be defined as the source brightness temperature corrected for atmospheric attenuation, radiative loss, and rearward and forward scattering and spillover if the source is equal to or larger than the main diffraction beam.

# C.3 Telescope Efficiency Measurements

Telescope efficiencies are normally calculated using a measurement of the continuum brightness of a planet (for $\eta_{mb}$) or the Moon (for $\eta_{fss}$). In the following I give the relations used to calculate several telescope efficiencies. Since the source coupling between a disk source like the planets and a Gaussian telescope beam is given by:

$$\eta_{cmb} = 1 - \exp\left[-\ln(2)\left(\frac{\theta_s}{\theta_B}\right)^2\right] \tag{C.20}$$

I will use this term in the efficiency equation derivations given below.

## C.3.1 Corrected Main Beam Efficiency

The efficiency factor which converts the 12m $T_R^*$ scale to the $T_{mb}$ scale is given by:

$$\begin{aligned} \eta_m^* &= \frac{\eta_{mb}}{\eta_l \eta_{fss}} \\ &= \frac{T_R^*}{T_{mb}} \\ &= \frac{T_R^*}{(T_R - T_{bg})\left\{1 - \exp\left[-\ln(2)\left(\frac{\theta_s}{\theta_B}\right)^2\right]\right\}} \end{aligned} \tag{C.21}$$

One can also calculate $\eta_m^*$ using the Ruze equation:

$$\eta_m^* = \left[1 + \frac{A_e\theta_e^2}{A_m\theta_m^2}\right]^{-1} \tag{C.22}$$

given that:

$$\theta_e = 2\sqrt{\ln(2)}\frac{\lambda}{\pi c_\sigma} \tag{C.23}$$

$$\theta_m = 1.22\frac{\lambda}{D} \tag{C.24}$$

$$\frac{A_e}{A_m} = \frac{1}{\eta_{a0}}\left[\frac{2c_\sigma}{D}\right]^2\left\{\exp(\delta^2) - 1\right\} \tag{C.25}$$

where $\lambda$ is the wavelength of observation, $c_\sigma$ is the correlation scale size of the surface deviations (280 mm), $\eta_{a0}$ is the zero wavelength aperture efficiency (0.55), and $\delta$ is the surface accuracy (75 $\mu$m). Figure C.1 shows this relation with the actual measurements given in Table C.1.

## C.3.2 Main Beam Efficiency

The efficiency factor which converts any source antenna measurement to the $T_{mb}$ scale is given by:

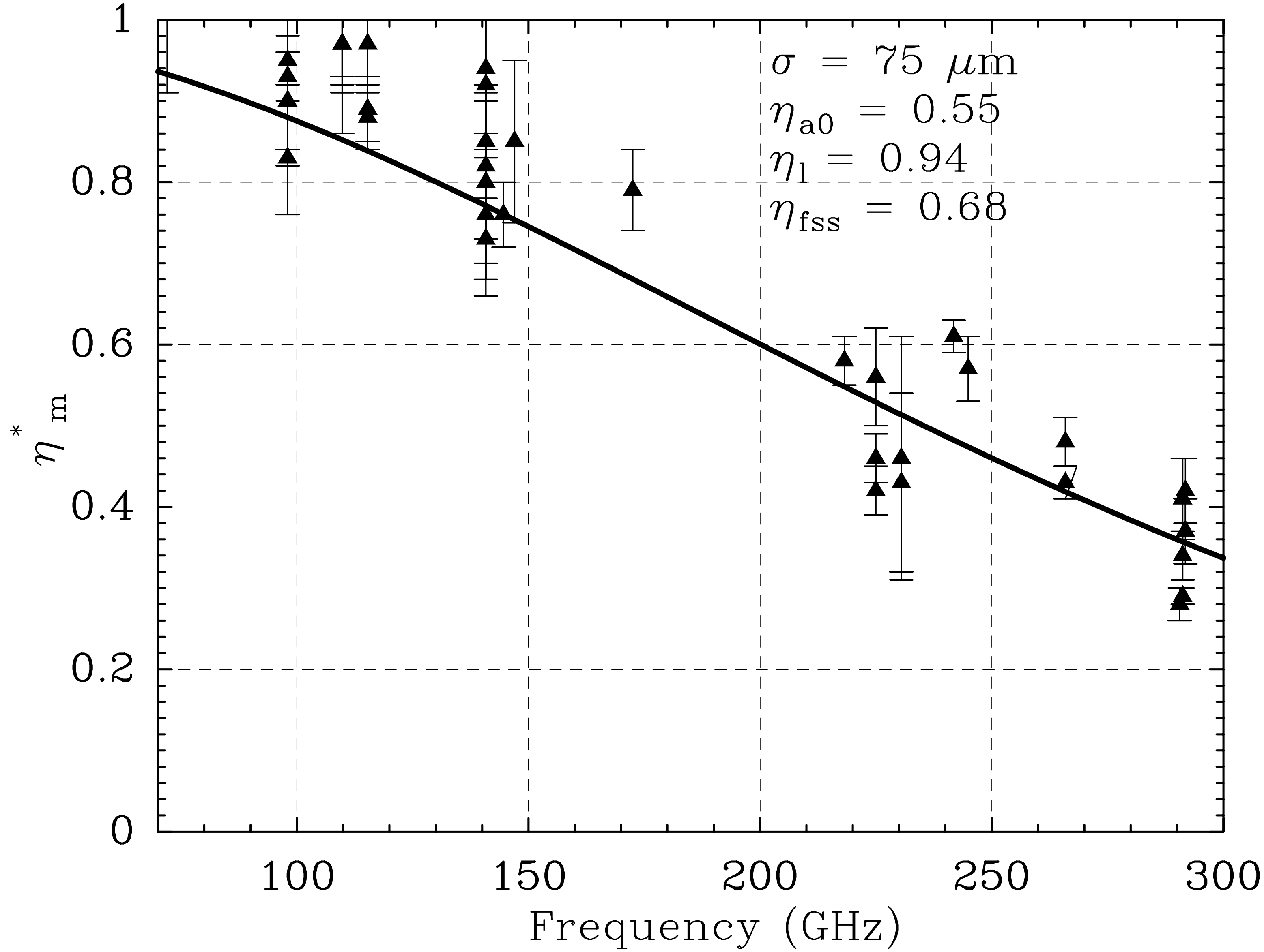


Figure C.1: Measured and theoretical estimates of $\eta_m^*$ as a function of frequency.

$$\begin{aligned} \eta_{mb} &= \frac{T'_A}{T_{mb}} \\ &= \frac{\eta_l \eta_{fss} T_R^*}{(T_R - T_{bg})\left\{1 - \exp\left[-\ln(2)\left(\frac{\theta_s}{\theta_B}\right)^2\right]\right\}} \end{aligned} \tag{C.26}$$

Table C.1 lists the most recent measurements of $\eta_m^*$ and $\eta_{mb}$ for the planets and frequencies given.

Table C.1: 12m Telescope Efficiencies

| Frequency (GHz) | Source | Source Size (arcsec) | $\eta_m^*$ | $\eta_{mb}$ | Date Measured |
|---|---|---|---|---|---|
| 72.0 | Jupiter | 37.4×35.0 | 1.04±0.13 | 0.66±0.08 | April 1997 |
| 98.0 | Venus | 12.2×12.2 | 0.90±0.06 | 0.57±0.04 | November 1996 |
| 98.0 | Mars | 8.5×8.4 | 0.83±0.07 | 0.53±0.04 | June 1997 |
| 98.0 | Saturn | 16.9×15.3 | 0.93±0.11 | 0.59±0.07 | June 1997 |
| 109.8 | Venus | 22.1×22.1 | 0.97±0.04 | 0.62±0.02 | August 1996 |
| 109.8 | Jupiter | 43.4×40.6 | 0.97±0.05 | 0.62±0.03 | August 1996 |
| 109.8 | Saturn | 19.2×17.3 | 0.97±0.06 | 0.62±0.04 | August 1996 |
| 115.3 | Jupiter | 43.4×40.6 | 0.88±0.04 | 0.56±0.02 | August 1996 |
| 115.3 | Saturn | 19.2×17.3 | 0.97±0.06 | 0.62±0.04 | August 1996 |
| 140.8 | Venus | 13.4×13.4 | 0.73±0.05 | 0.47±0.03 | November 1996 |
| 140.8 | Mars | 5.5×5.5 | 0.85±0.07 | 0.54±0.04 | November 1996 |
| 140.8 | Jupiter | 35.5×33.2 | 0.82±0.09 | 0.52±0.06 | November 1996 |
| 140.8 | Saturn | 19.1×17.2 | 0.92±0.09 | 0.59±0.06 | November 1996 |
| 140.8 | Uranus | 3.5×3.5 | 0.80±0.10 | 0.51±0.06 | November 1996 |
| 140.8 | Neptune | 2.2×2.2 | 0.76±0.10 | 0.48±0.06 | November 1996 |
| 144.6 | Saturn | 17.4×15.7 | 0.76±0.04 | 0.48±0.03 | June 1996 |
| 218.2 | Saturn | 18.0×16.1 | 0.58±0.03 | 0.37±0.02 | November 1995 |
| 230.5 | Saturn | 19.5×17.6 | 0.43±0.11 | 0.27±0.07 | October 1996 |
| 230.5 | Uranus | 3.6×3.5 | 0.46±0.15 | 0.29±0.09 | October 1996 |
| 244.9 | Mars | 7.5×7.5 | 0.57±0.04 | 0.36±0.02 | December 1996 |
| 265.9 | Mars | 5.5×5.5 | 0.48±0.03 | 0.31±0.02 | November 1996 |
| 291.8 | Mars | 7.1×7.1 | 0.42±0.04 | 0.27±0.02 | December 1996 |
| 291.8 | Saturn | 17.8×16.1 | 0.37±0.04 | 0.24±0.02 | December 1996 |