Appendix E

Walsh Function Modulation

All observations which involve position switching or focus modulation (which is usually only done with frequency switching measurements) at the 12 Meter Telescope use a Walsh function cycle for each SIG–REF measurement. For a given measurement of duration P composed of n SIG–REF measurement pairs (a SIG–REF pair is referred to a "repeat" in the 12m telescope lingo), a switching sequence determined by the Walsh function of PALey order $2^n - 1$ is built-up. Building up a PALey order of $2^n - 1$ Walsh functions, where n is the number of ON–OFF pairs, perfectly rejects polynomial drift terms of order up to and including $n - 1$. This statement implies that...

$$\int_0^P \left(\sum_1^{n-1} a_{n-1} t^{n-1} \right) PAL(2^n - 1, t) dt = 0 \tag{E.1}$$

This sequence of SIG–REF switching can be truncated at any even point. Ideally it would be trucated after exactly 2^n phases, at which point it gives the maximum rejection of an t^{n-1} and lower order polynomial drift. Although we recommend that observers try to truncate this way, we don't insist on it. The order of polynomial drift completely rejected is then a function of the biggest 2^m sequence (or the inverse of this sequence) that has been repeated without truncation. Since the PALey order is calculated in real time, we don't need to know in advance how many terms there will be. The algorithm is simply the following...

> After every complete set of m (2 or more, but always a power of 2) phases, the next m phases will be a repeat of the first m, with 0 and 1 reversed.

This is equivalent to adding another order of Rademacher function to the R products generating a Walsh function of order $2^n - 1$

$$\begin{aligned} PAL(2^n - 1, t) &= \prod_1^n R(n, t) \\ &= \prod_1^n \left\{ sign \left[\sin(2^n \pi t) \right] \right\} \end{aligned} \tag{E.2}$$

This assures we build up a PALey order of ($2^n - 1$,t), with n being increased by 1 after every complete set, and which means that polynomial drift terms up to order $(n-1)$ are perfectly rejected.

In practice each phase usually lasts 30 seconds, although phases as short as 10 seconds and as long as a minute are occasionally used. How long a sequence we have really depends on how long the observer wants to integrate. The "0 1 1 0" is normally the shortest sequence, with the 2 pairs of phases. We probably rarely use more than 8 pairs, which would be an 8 minute (in total time) integration at 30 seconds per SIG and REF integration phase. In other words, typical functions are usually PAL(3,T), PAL(7,T) or PAL(15,T). Figure E.1 shows the Walsh functions of order $2^n - 1$ for $n = 1 - 5$.

The following C code fragment (written by Jeff Hagen) is used in the 12 Meter Telescope control program to calculate the Walsh function.

```
walsh(i)
int i;
{
  int bool = 0;
  int mask = 0x8000;

  while(mask) {
    if( mask & i )
      bool += 1;
    mask = mask>>1;
  }
  return( bool % 2 );
}
```

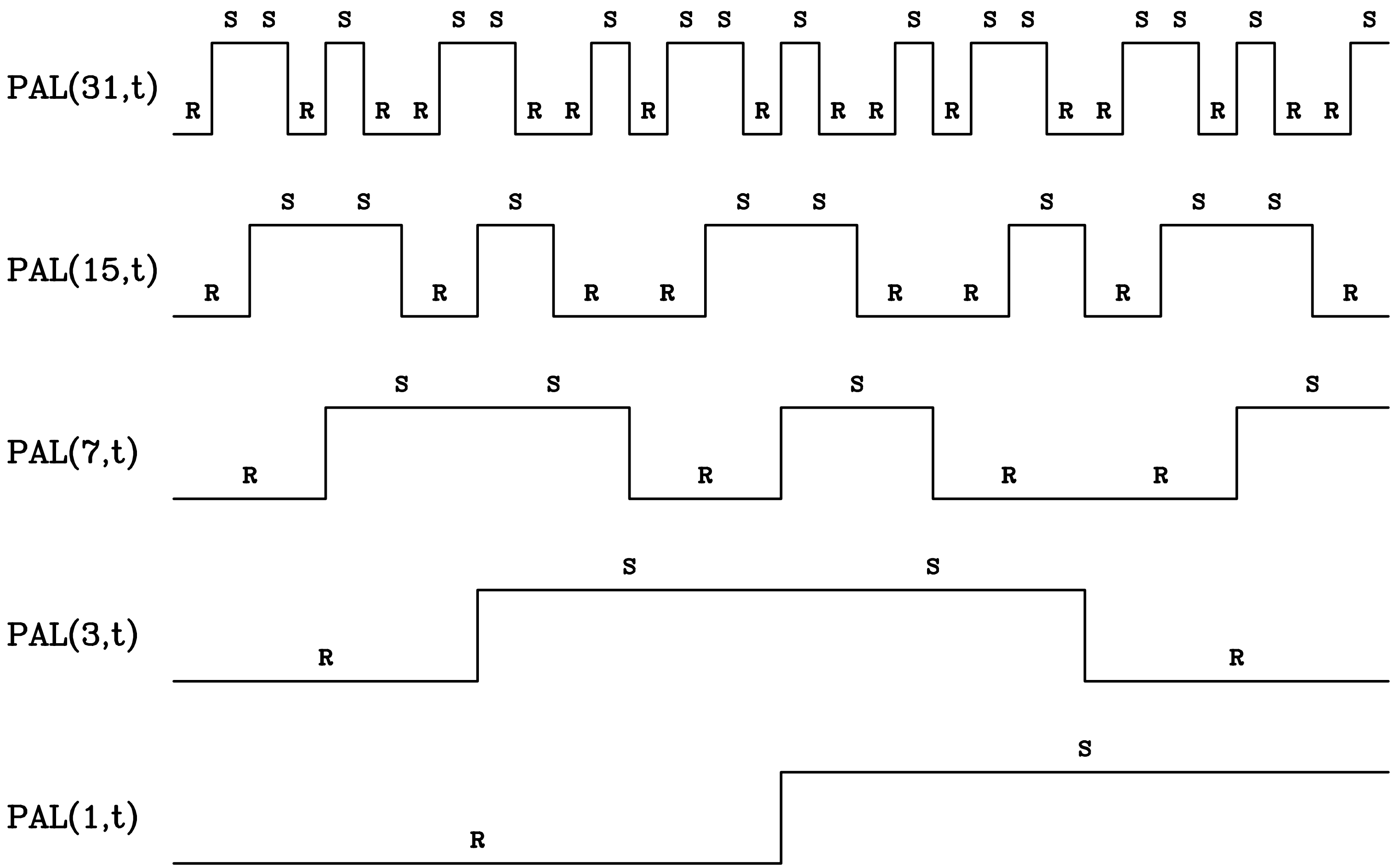

Figure E.1: The first five Walsh functions used in switched observations at the 12m telescope. The order of the Walsh function PAL is given by $2^n - 1$.