The characteristic strain spectrum:

$$h_c(f) = A\left(\frac{f}{f_{ref}}\right)^{\alpha} \qquad h_c(f) = \sqrt{f P_h(f)}$$

The Power spectrum of the induced timing residuals:

$$P_r(f) = \frac{P_h(f)}{12\pi^2 f^2} \qquad P_r(f) = \frac{h_c^2(f)}{12\pi^2 f^3}$$

f = gravitational wave frequency
f_{ref} = reference frequency (typically 1/(1 yr)))
Δt = average time between observations
A = Amplitude of the GW background
T = total observing time
N = number of data points per pulsar
M = number of pulsar pairs

α= -2/3 for Black hole binary background
α = -1 for Ω(f) = constant.

$$\Omega(f) = \frac{2}{3}\frac{\pi^2}{H_0^2} f^2 h_c(f)^2 \qquad P_r(f) = \frac{H_0^2}{8\pi^4}\frac{\Omega(f)}{f^5}$$

The noise power is assumed white:

$$P_n(f) = 2\sigma_n^2 \Delta t$$

Simple scaling relationships may be derived for two limiting cases:

$$P_r(f) << P_n(f) \qquad P_r(f) >> P_n(f)$$

$$h_c(f_l) << h_0 \qquad h_0 = \sqrt{\frac{24\pi^2}{N}\frac{\sigma_n}{T}} \qquad h_c(f_l) >> h_0$$

$$SNR_l \propto \frac{A^2}{\sigma_n^2}\sqrt{M(M-1)}NT^{-2\alpha+2} \qquad SNR_H \propto \sqrt{N}\sqrt{M(M-1)}$$

T = total observing time M= number of pulsars N = number of data points

For T = 5 yr, N=250, M = 20, σ_n = 100 ns: $h_0 = 10^{-15}$

For A = 10^{-16}, SNR_l=.2
For A>>10^{-16}, SNR_H=12

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