Estimate of FASR Requirements for CME Detection below 300 MHz

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December 20, 2004

1. Introduction

A tentative goal of the Frequency Agile Solar Radiotelescope (FASR) is to measure the spatial, temporal, and frequency structure of coronal mass ejections (CME's) as they expand away from the surface of the sun. This memo estimates the required collecting area of FASR below 300 MHz for a prototypical CME based on observations by Bastian *et al* (2001) between 164 and 432 MHz with the radioheliograph at Nancay, France. Out to a distance of about one solar radius from the photosphere the CME spectrum drops sharply below 150 MHz, presumably due to Razin-Tsytovich suppression, but at greater distances from the sun the CME loop should be detectable as low as 50 MHz at brightness temperatures greater than 10,000 K or so. The Nancay data show that the spectral index of the CME loop decreases very rapidly with increasing solar radius and the radiation model proposed infers that measurements below 150 MHz will yield considerable information about the CME emission mechanisms within a few solar radii.

For the calculations in this memo I will assume a 3-σ surface brightness detection goal of 5000 Kelvins. Coronal scattering is likely to set a lower limit on source spatial scale of about 3 arcminutes at 100 MHz, but this scattering size will decrease rapidly with solar radius and increase roughly in proportion to observing wavelength. Hence, example calculations will be shown for FASR array sizes of 1.5, 3, and 6 km.

Thanks to Dale Gary for a number of helpful suggestions.

2. Basic Equations

Sidelobe confusion of the synthesized beam (map dynamic range) needs to be carefully analyzed in connection with CME mapping, but this memo is only concerned with array sensitivity to an isolated patch of sky. Hence, I shall assume a uniform brightness thermal source whose solid angle is equal to the solid angle of the synthesized beam. The relationship between beam solid angle, brightness temperature, and source flux density in Watts per square meter per Hertz is

$$S = \frac{2kT_b\Omega}{\lambda^2}, \tag{1}$$

where k is Boltzmann's constant (1.38×10^{-23} joules/Kelvin), T_b is the source brightness temperature, Ω is the beam or source solid angle in steradians, and λ is the wavelength in meters. Converting to flux density in Janskys and frequency, f, in MHz Equation 1 becomes

$$S = 0.0307 T_b \Omega f^2 \tag{2}$$

For example, if $T_b = 5000$ K, $f = 100$ MHz, and $\Omega = 6.0x10^{-7}$ (3 arcminute beam), then $S = 0.92$ Jy.

If L is the longest array baseline, then the synthesized beamwidth will be approximately

$$\theta \cong \frac{\lambda}{L} \tag{3}$$

and

$$\Omega \cong \frac{\pi\theta^2}{4} = \frac{\pi\lambda^2}{4L^2} = \frac{\pi 300^2}{4f^2L^2} \tag{4}$$

Combining Equations 2 and 4 we get

$$S \cong \frac{2170 T_b}{L^2} \tag{5}$$

If $T_b = 5000$ K and $L = 3000$ m, then $S \cong 1.0$ Jy.

The 1-σ equivalent flux density thermal noise from one polarization of an array with a total effective collecting area of A_e and system temperature T_{sys} is (Crane & Napier 1989)

$$\Delta S = \frac{2kT_{sys}}{A_e\sqrt{B\tau}} \tag{6}$$

where B is the pre-correlation bandwidth in Hertz, and τ is the integration time in seconds. If we let ΔS equal the source flux density in Equation 5, then a 3-σ detection will require an effective total collecting area of the array of

$$A_e = \frac{6kT_{sys}}{S\sqrt{B\tau}} = \frac{8280 T_{sys}}{S(Jy)\sqrt{B\tau}} = \frac{3.186 T_{sys} L^2}{T_b\sqrt{B\tau}} \tag{7}$$

If $T_{sys} = 1000K$, $S = 1.0$ Jy, $B = 1$ MHz, and $\tau = 60$ sec, then $A_e = 1069$ m^2. A typical CME expansion speed is roughly one arcminute per minute so integration times of more than 60 seconds can often be employed without smearing the spacial structure significantly. Since the pixel size is proportional to wavelength, longer integration times can be used at lower frequencies, which partially compensates for the higher system temperatures at longer wavelengths.

3. Results for Three Array Sizes

We can now use the equations in the previous section to compute the number of array elements required for a chosen set of array parameters and source surface brightness. Tables 1, 2, and 3 show the results for three arrays of 1.5, 3, and 6 kilometer size for the following assumptions:

$$\begin{aligned} T_{sys} &= 200 + 10^3 (f(MHz)/100)^{-2.7} \;\; Kelvins \\ T_b &= 5000 \;\; Kelvins \\ B &= 1 \;\; MHz \\ \tau &= 60 \frac{100}{f(MHz)} \frac{3000}{L} \;\; seconds \end{aligned}$$

The numbers of array elements are computed for antenna gains four times that of an isotropic antenna (+6 dBi), which is typical of the broadside gain of a dipole over a ground screen. The equivalent numbers of parabolic dishes are computed for ten-meter diameter antennas with 50% aperture efficiency. Ten-meter dishes will be less than four wavelengths across below 120 MHz so numbers of dishes are not listed below this frequency.

REFERENCES

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This preprint was prepared with the AAS LaTeX macros v5.2.

Table 1: Required numbers of array elements or array dishes as a function of frequency for an array of 1500-meter maximum baseline length using the assumptions stated at the beginning of section 3.

Freq. MHz	T_{sys} K	Resol'n arcmin	Int Time sec	Flux Dens Jy	A_e m^2	# Elem (6 dBi)	# Dishes (10-m)
10	501387	68.75	1200	4.82	24879	87	
20	77329	34.38	600	4.82	5426	76	
30	26009	22.92	400	4.82	2235	71	
50	6698	13.75	240	4.82	743	65	
75	2374	9.17	160	4.82	323	64	
88	1612	7.81	136	4.82	237	65	
120	811	5.73	100	4.82	139	71	4
150	535	4.58	80	4.82	103	81	3
200	354	3.44	60	4.82	79	110	2
250	284	2.75	48	4.82	71	154	2
300	251	2.29	40	4.82	68	215	2

Table 2: Required numbers of array elements or array dishes as a function of frequency for an array of 3000-meter maximum baseline length using the assumptions stated at the beginning of section 3.

Freq. MHz	T_{sys} K	Resol'n arcmin	Int Time sec	Flux Dens Jy	A_e m^2	# Elem (6 dBi)	# Dishes (10-m)
10	501387	34.38	600	1.20	140735	492	
20	77329	17.19	300	1.20	30696	429	
30	26009	11.46	200	1.20	12645	398	
50	6698	6.88	120	1.20	4204	367	
75	2374	4.58	80	1.20	1825	359	
88	1612	3.91	68	1.20	1342	363	
120	811	2.86	50	1.20	789	397	21
150	535	2.29	40	1.20	581	457	15
200	354	1.72	30	1.20	444	621	12
250	284	1.38	24	1.20	399	871	11
300	251	1.15	20	1.20	387	1215	10

Table 3: Required numbers of array elements or array dishes as a function of frequency for an array of 6000-meter maximum baseline length using the assumptions stated at the beginning of section 3.

Freq.	T_{sys}	Resol'n	Int Time	Flux Dens	A_e	# Elem	# Dishes
MHz	K	arcmin	sec	Jy	m^2	(6 dBi)	(10-m)
10	501387	17.19	300	0.30	796117	2779	
20	77329	8.59	150	0.30	173645	2425	
30	26009	5.73	100	0.30	71530	2248	
50	6698	3.44	60	0.30	23781	2076	
75	2374	2.29	40	0.30	10325	2028	
88	1612	1.95	34	0.30	7594	2053	
120	811	1.43	25	0.30	4462	2243	114
150	535	1.15	20	0.30	3288	2583	84
200	354	0.86	15	0.30	2513	3509	64
250	284	0.69	12	0.30	2257	4924	58
300	251	0.57	10	0.30	2187	6872	56