A Rough Estimate of Mutual Coupling Effects

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1. Introduction

One of the key issues in the design of the low frequency FASR array and the Long Wavelength Array (LWA) is the effect of mutual coupling on array performance. At the Santa Fe meeting (http://lwa.unm.edu/events_pubs/) Bill Erickson pointed out that this can be a serious problem, if not dealt with carefully. This memo offers a simplified view of mutual coupling that might aid the design discussions of these two instruments. Some of the thoughts written here are conjectural and intended to stimulate feedback and corrections from others who may have a more detailed understanding. Thanks to Steve Ellingson for some very helpful comments.

Consider an array that is connected to a transmission line network with perfect isolation between the transmission line ports that are connected to the array. In other words, the only coupling between array elements is through free space.

Mutual coupling has at least two important effects on an array's performance. One is that the far-field pattern of a single array element is different when it is embedded in the array from what it is in isolation. The other effect is that the radiation impedance of an element is different in the array than it is in isolation. Both effects are due to scattering of radiation from neighboring elements. From the transmitting point of view, some energy reflects from adjacent elements back into the transmitting element. This reflected energy affects the element's effective impedance in the same way that a reflection on a transmission line affects the measured impedance at the line's input terminals. The scattered radiation from adjacent elements also adds vectorially with direct radiation in the far field, thus modifying the effective far-field pattern of an element. The efficiency of one element in a radio astronomy array for a chosen direction in the sky can be thought of as the product of its far-field response in that direction and the power transfer coefficient from the element to its amplifier that may be less than one due to an impedance mismatch.

In an infinite array, the embedded element far field pattern and the array factor are factorable. That is to say, the array's far field pattern is the product of the array factor and the embedded element far-field pattern. The array factor is computed assuming isolated, isotropic radiators and the chosen phase and amplitude weights. Life is a bit more complicated in a finite array where the border elements will have different embedded patterns that affect their relative weighting in the array factor summations. Nevertheless, the concept of embedded element pattern is still very useful in computing the array's far-field response.

It is a difficult, but not impossible task to design an efficient array that has strong coupling coefficients between elements, e.g., the University of Massachusetts Vivaldi array (Shin & Schaubert 1999). However, for the moment we are only concerned with the effects of coupling on array's performance, given the far-field pattern and terminal impedance of each embedded element. These quantities can either be measured or computed with a modern antenna analysis program. The following will illustrate a few examples using typical coupling coefficients.

Two readable references on mutual coupling are section 3.6 of Stutzman & Thiele (1998) and Pozar (1994).

2. Far-Field Element Patterns

The simplest array contains two elements. Let each be terminated by a matched load, and let C be the complex coupling coefficient between the two elements that can be determined by measuring the amplitude and phase of the current induced in one element by power transmitted from the other element. If E_o is the relative field strength in a given direction in the sky for a single, isolated element, then the field strength for one element in the array pair, E, will be

$$E = E_o(1 + Ce^{i2\pi\delta/\lambda}), \tag{1}$$

where δ is the path length difference to the far-field direction from the two elements, and λ is the wavelength. If the magnitude of the coupling coefficient is -20 dB ($|C| = 0.1$), then the extremes of E/E_o will be 0.9 and 1.1, or -0.9 and +0.8 dB.

If there are n array elements in the vicinity of the element for which we are computing a far-field pattern, then the pattern in the presence of mutual coupling will be

$$E = E_o(1 + \sum_n C_n e^{i2\pi\delta_n/\lambda}), \tag{2}$$

If there are four adjacent elements in the array (five total), each with a coupling coefficient of -20 dB, then the extremes of E/E_o will be 0.6 and 1.4, or -4.4 and +2.9 dB. This is quite substantial.

Figure 2 shows the H-plane, far-field patterns of an isolated dipole and the same dipole paired with one and two identical dipoles parallel to it. The curves are computed with a simple model of point source radiators over an infinite ground plane using an excitation of the passive elements equal in phase and relative amplitude to the coupling coefficients. The coupling coefficients were computed with the CST antenna analysis program for sleeved dipoles whose driven element was 0.2 wavelengths above an infinite ground plane. (More information on the CST software package, calculations, and sleeved dipole will be presented in a later memo.) The same sleeved dipole calculations provided the H-plane, far-field patterns, which are shown as symbols in Figure 2. Agreement between rigorous calculations and a point-source radiator model with coupling is remarkably good, which gives some credence to the simple picture outlined here. To get a good match between the isolated dipole patterns computed with the point-source model and with CST, the point sources needed to be slightly closer to the ground plane than the center of the sleeved dipole, which may indicate that the radiating region of the sleeved at the chosen frequency may be a bit closer to the ground plane than the driven element. The sleeved-dipole phase center computed by CST is 0.18 wavelengths above the ground plane.

As one adds more elements to the array the new elements will be more distant from the element under investigation so the coupling can be expected to be weaker. However, it is sobering to realize that, if the coupling coefficient does not decrease faster than $1/r$ ($1/r^2$ power decrease), the summation in Equation 2 may not converge because the number of elements per unit distance in a uniform, planar array will increase in proportion to r. Typically, the coupling will decrease faster than $1/r$, but there is no guarantee that it will. There are practical arrays which can have nulls produced in their far-field response by mutual coupling, and Equation 2 suggests that the coupling between elements does not have to be terribly strong for this to happen.

3. Active and Passive Impedances

There are two common definitions of impedance associated with elements embedded in an array found in the literature, “active” and “passive.” Passive impedance is the impedance

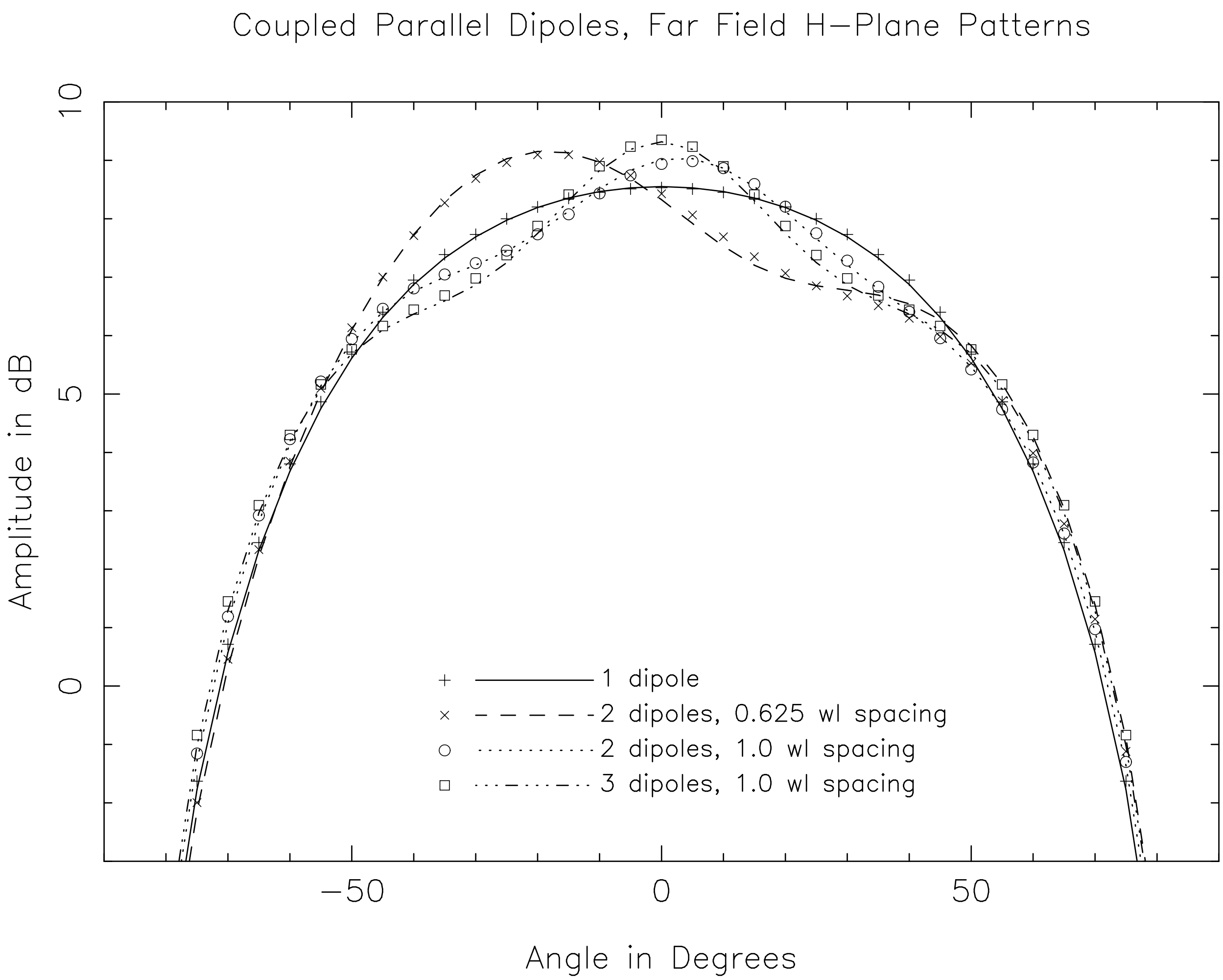

Fig. 1.— Far field radiation H-plane patterns for a dipole 0.16 wavelengths above an infinite ground plane. The solid curve shows the pattern of an isolated dipole. The dashed curve shows the pattern with one passive, parallel dipole 5/8-wavelengths away with a coupling coefficient of -17.7 dB as computed with the CST antenna analysis package. The dotted curve is also with two dipoles but spaced one wavelength apart and a coupling coefficient of -24.5 dB. The dash-dot-dot-dot curve shows the effect of adding a second passive element one wavelength on the other side of the driven element. The symbols show the patterns computed by CST for the same configurations.

that is measured at the terminals of an array element with all of the other elements terminated in their load impedance. Active impedance applies to a transmitting array with the elements phased to transmit in a given direction or to a receiving array illuminated by a far-field source in that same direction.

The transmitting array assumes that all elements are driven with portions of the transmitted power. Hence, the return signal, normally associated with an impedance mismatch, seen at a given element port includes the signals radiated from adjacent elements and coupled into the element under measurement by mutual coupling. The phase of these coupled signals depend on the direction to which the transmitting beam is steered, and the resulting active impedance also depends of beam steering direction. There would seem to be a lack of reciprocity between transmitting and receiving arrays because a single element in a receiving array knows nothing about the direction to which the array's receiving beam is phased. This apparent paradox is only the result of the differences in impedance definition.

Consider the case where mutual coupling has conspired to create a null in the array's response in a certain direction. A calculation of the active impedance at each array element for that phasing direction will show that all of the transmitted power is reflected back into the transmitter. In other words, each element will appear as a complete impedance mismatch to the transmission line connected to it. In the receive case, radiation from a radio source in the direction of the array null arrives at an array element directly from the source and via scattering from adjacent, mutually coupled elements. The sum of the scattered energy will exactly cancel the direct radiation. Hence the array presents a complete impedance mismatch to the sky in the null direction, and all radiation arriving from that direction will be reflected back into the sky. The receiving mismatch is harder to express as an intrinsic impedance property of array elements, so passive impedance must be combined with an analysis of the scattered signal summation to derive the array's receive response in a given direction. Reciprocity still holds. (A preamp with the appropriate input impedance can be added to each element of a receive-only array without affecting the validity of the analysis of the array's properties.) If an array is analyzed as a transmitting array, its far field gain pattern applies to the receive case, too.

An important consequence of mutual coupling is that it can cause a poor impedance match of the array to the sky in certain directions. This is a real signal loss due to reflection at each array element, and this power is not available to the element preamp. Hence, the signal-to-noise ratio at each element is diminished. No amount of signal processing can compensate for this SNR loss.

The passive impedance of an array element is a useful concept in the receive case because it can be used with the preamp's input impedance to calculate how much of the power that

is available is transferred to the preamp. A simple picture of the effect of mutual coupling on the passive impedance of an array element is to assume that the measured return loss from scattering off one adjacent element is twice the coupling coefficient in dB. In other words, if a signal is transmitted from element 1, it will induce a current in an adjacent element 2 in proportion to the coupling coefficient, and element 2, in turn, will return the same fraction of its induced power to element 1. For example, if the coupling coefficient is -20 dB, the induced current in element 2 will be 10% of the current in element 1. Element 2 will then induce 10% of its current back into element 1 for a total return loss of 40 dB.

Mutually coupled currents will add vectorially according to their round-trip phase delays so that, in the worst case, two elements with -20 dB coupling coefficients will produce a return loss of 34 dB and four elements will produce a return loss of 28 dB. If the coupling coefficient were -15 db, the worst-case return loss for one, two, and four coupled elements would be 30, 24, and 18 dB, respectively.

4. What Affects Mutual Coupling

Thus far we have assumed that all of the elements of an array are terminated in their characteristic impedance. If a coupled element is open-circuited (termination load removed), it will conduct very little induce current and, hence, will produce a much smaller effect on the pattern and impedance of adjacent elements. Therefore, if the array elements are lightly coupled to their amplifiers, the effects of mutual coupling are reduced. In practice, this is of practical use only if the system noise temperature is dominated by sky noise. Otherwise, the array's sensitivity will be degraded. In principle, since the termination impedance of array elements affects mutual coupling, it is possible to solve for termination impedances that minimize the effects of coupling while retaining acceptable sensitivity, but this is a complex procedure, particularly for wide bandwidths and scan angles, and possibly not guaranteed to have a practical solution.

Collinear dipoles couple less strongly than parallel dipoles for a given center-to-center spacing, but the difference is generally less than 10 dB. Element support structure can affect mutual coupling so that dipoles on metal posts can have a different coupling coefficient than ideal dipoles in free space. Circularly polarized elements of the same polarization tend to have low mutual coupling, but oppositely circularly polarized elements do couple as strongly as linear elements. Hence, a dual circularly polarized array will have significant coupling between polarizations.

At first thought, directive array elements, such as yagis, would seem to have less mutual coupling for a given spacing, but the opposite is often the case. Far-field intuition doesn't necessarily apply to near-field effects. A qualitative way to understand this is that more directive antennas have larger collecting areas which will begin to overlap at larger spacings.

If the array elements are not perfectly isolated from the transmission line system, then reflections from mismatches and imperfect isolation in power combiners will add to the effects of mutual coupling. This can create a very complex and frequency dependent array analysis problem and compromise array performance.

5. Amplifier Noise Considerations

Noise generated in the amplifiers tied to the array elements will enter into the mutual coupling problem in at least two ways. First, noise will be broadcast from the input terminals of the amplifiers to adjacent elements and produce partially coherent noise in the array. Exactly how this noise manifests itself in the array signal processing algorithms can be important. Second, signal combining techniques for compensating for differences in mutual coupling at the borders of an array will often be only partially effective in post-amplifier signal processing because the incoherent noise from the amplifiers will be unevenly weighted and cause signal-to-noise ratio degradation from an ideal array. If the array system temperature is dominated by sky noise, this is of less concern.

Below about 100 MHz, where sky noise dominates the system temperature, one has more latitude in selecting load impedances on the array to modify the mutual coupling coefficients for better for a field response. Modest power loss due to a mismatch between a preamp and its element will not raise the system temperature significantly.

6. Ongoing Work

In a companion memo to follow we will present a circuit model and associated matrix representation of mutual coupling that accounts for all effects of mutual coupling and offers a method for fully characterizing an array through a reasonably small set of measurements or simulated antenna calculations.

REFERENCES

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